Yang-Mills Dynamics as an Effective Theory of Composite Vector Bosons.

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Summary. — Alternatively to the evaluation of path integrals, weak-mapping theorems of quantum fields can be used for the derivation of relativistic composite particle dynamics from a given dynamics of subcomponent fields (particles). These theorems are applied to a nonlinear spinor-isospinor field with nonperturbative Pauli-Villars regularization and canonical quantization, where vector boson states were calculated as two-fermion (spinor-isospinor) composites. In the low-energy limit the weak mapping of the functional equation of this quantized spinor field leads to the field-theoretic functional equations for a Yang-Mills field which governs the dynamics of the composite vector bosons. In the framework of these theorems the quantum properties of this mapped SU(2) field are also derived.

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Introduction.

The question whether observed forces and particles are elementary or whether they are due to a subcomponent structure is basic in any atomistic model of matter. The standard model of matter at least contains 37 «elementary» particles and more than 20 parameters. This strongly supports the assumption that this model describes composite particles and resulting forces rather than elementary ones. But if one assumes that compositeness is involved, the next basic problem is to show how the standard model can be deduced from a postulated subcomponent dynamics. Generally this is the problem of deriving effective actions for composite particles from a given subcomponent theory. Formulating elementary-particle dynamics by quantum field theory, in this framework composite particles have been described by field operator products. The derivation of an effective dynamics for field operator products from the original field can be characterized as «strong» mapping between quantum fields. After various attempts to realize such strong mappings path integrals are used at present to solve this problem. However, from the conceptional point of view as well as from the technical point of view this procedure has serious drawbacks and difficulties which so far have not been removed [1]. In order to avoid these difficulties
an alternative program has been started by one of the authors which can be referred to as weak mapping of quantum fields [2].

Quantum field theories can be characterized by functional equations. In contrast with the strong mapping by means of operator products in the weak mapping procedure functional equations are mapped onto effective functional equations in order to obtain an effective composite-particle dynamics. These weak mappings respect the algebraic properties of quantum fields and are free from the difficulties and insufficiencies of strong mappings, provided the basic quantum field theories are sufficiently regularized.

The weak mapping has been exemplified for basic spinor fields with four-fermion interactions and nonperturbative Pauli-Villars regularization [3]. As a first step to explain the standard model on a composite-particle basis one can use this spinor model to derive the Yang-Mills dynamics by the action of composite vector bosons. This will be done in this paper. It is based on a preceding paper by one of the authors [4] where a first proof of our assumption was already given. With respect to this paper the following improvements have been incorporated.

In the preceding paper [4] the weak mapping was performed by elementary functional calculation rules. In the meanwhile, exact weak-mapping theorems were derived [5] which replace and simultaneously justify these calculational rules. In addition, a refined calculation of [4] was performed by one of the authors [6], based on exactly calculated composite vector boson states [7]. Furthermore, the problem of discovering the quantum rules on the composite-particle level is discussed which was not treated in [4]. Finally, as the weak mapping leads to a Yang-Mills dynamics in the low-energy limit, the estimates and calculations of additional terms which vanish in that limit are performed or improved, respectively. So a renewed treatment of this problem giving more detailed and refined information is justified.

Without reference to [4] Suzuki later on proposed a similar model [8] and claimed to have derived Yang-Mills dynamics as a composite-particle effect. His claim is based on analysing scattering cross-sections for composite vector bosons and discovering Yang-Mills boson scattering. This supports the results derived in [4] and in this paper. However, the comparison of cross-sections cannot compete with or even replace the analytical tool of weak-mapping theorems which lead to the conclusions of [4] and of this paper.

For abbreviation in the following a condensed notation is used:

\[ \partial_\mu := \frac{\partial}{\partial x^\mu} \] for Greek small letter index, \( \mu = 0, 1, 2, 3 \),

\[ \nabla_s := \frac{\partial}{\partial x^s} \] for \( s = 1, 2, 3 \) space coordinates,

\[ \partial_l := \frac{\partial}{\partial j_l} \] for Latin capital letter index,

\[ \partial_k := \frac{\partial}{\partial b_k} \] for Latin small letter index,

\( \{A_1 \ldots A_n\} := \) antisymmetrization of \( A_1 \ldots A_n \) if not otherwise stated; in contrast \( \{a_n\} \) means the set \( a_1 \ldots a_n \) etc.

The summation convention is used throughout the paper.