On the Application of Wavelet Analysis to Separation of Secondary Particles from Nucleus-Nucleus Interactions.

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Summary. — We apply wavelet analysis for separation of secondary particles from different channels of the $d + Au \rightarrow \ldots$ reaction. Using the data obtained in the experiments at Nuclotron (Dubna) in March 1994 at a deuteron momentum of $3.8\,\text{GeV}/c$, we found 4 different regions in the time-of-flight vs. energy loss plot.


1. – Introduction.

The simplest way to tackle with almost any physical problem is to build a functional basis with the same symmetry as that of the original problem or close to it. That is why the Bessel functions fit the problems with cylindrical symmetry, as well as spherical functions fit the $SO_3$ symmetrical ones. The application of this very idea to certain classes of stochastic processes found its implementation in the wavelet analysis.

The Brownian motion

$$P(X(t) - X(t_0)) = \frac{1}{\sqrt{4\pi D |t - t_0|}} \exp \left( - \frac{(X(t) - X(t_0))^2}{4 D |t - t_0|} \right)$$

— one of the most common random processes — has long been known to be invariant under the scaling transformation (see, e.g., [1] for details)

$$P(b^{1/2}[X(bt) - X(bt_0)]) = b^{-1/2} P(X(t) - X(t_0)).$$

Therefore, it seems quite natural to use decomposition with respect to the affine group:

$$t' = \frac{t + b}{l},$$
namely dilatations and translations, when studying Brownian motion, as well as other similar random processes.

Technically, decomposition is performed by convolution of the function \( \psi \) with a certain function \( g(t) \), called wavelet, with the argument shifted to \( b \) and dilated by \( l \). It is essential that function \( g \) has limit supporter. Therefore, unlike the Fourier transform, which is inherently nonlocal, wavelet analysis or synthesis can be performed locally on a signal (field).

Based on the affine group representation [2], wavelet analysis and synthesis allow one to unfold a signal (a field), into space, time and direction. It works as a «microscope» discriminating different scales and a polarizer separating different angular contributions. Wavelet analysis has been applied to signal processing, image coding, turbulence data analysis and some other fields.

The numerous applications of wavelets to random data analysis (see, e.g., [3] and references therein) have proved them to be a powerful tool for studying fractal signals and data on cascading processes.

2. – Definitions.

As a decomposition based on an affine group

\[
\mathbf{x} \to a \mathbf{x} + b,
\]

the wavelet transform (WT) of an arbitrary function \( f(x) \) can be written as

\[
|f\rangle = \int |a, b; g\rangle \, d\mu(a, b)\langle a, b; g|f\rangle,
\]

where

\[
\langle l, b; g|f\rangle \equiv T_g(l, b, \theta) = C_g^{-1} \int_{R^n} f(x) g^* \left[ \Omega^{-1} (\theta) \frac{x - b}{l} \right] l^{-n} \, d^n x,
\]

\[
C_g = (2\pi)^n \int_{R^n} |\widehat{g}(k)|^2 \frac{d^n k}{|k|^n},
\]

\[
\widehat{g}(k) = (2\pi)^{-n} \int_{R^n} g(x) \exp[-ikx] \, d^n x.
\]

The rotation tensor \( \Omega \) belongs to the group \( SO_n \) rotations in \( R^n \) and depends on the Euler angles \( \theta \). In terms of the Euler angles \( \theta \) and scale (length) \( l \) the reconstruction formula (3) takes the form

\[
f(x) = \int_0^\infty \frac{dl}{l^{n+1}} \int_{R^n} g \left\{ \frac{x - b}{l} \right\} T_g(l, b, \theta) \, d^n b \, d\mu(\theta),
\]