Comments on the Vacuum Stability Bounds 
in the Case of Composite Higgs Bosons (*).

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Summary. — In case of one or two Higgs doublets the requirement of the vacuum stability establishes severe bounds between the heaviest Higgs boson mass and the top-quark mass. We argue that for the composite Higgs boson the standard arguments based on the vacuum stability requirement are not applicable and hence we cannot use the stability bound for the top-quark mass.

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1. — Introduction.

It has been proposed recently [1-3] that the electroweak symmetry breaking in the Weinberg-Salam model has a dynamical nature and it arises due to nonzero top-quark condensate \( \langle tt \rangle \neq 0 \). The Higgs boson in such scenario has a composite nature since there is no kinetic term for the Higgs field in the Lagrangian. In ref.[4] it has been demonstrated on the example of a toy model using the \( 1/N \)-perturbation theory that the model with composite Higgs is equivalent from the perturbative point of view to the corresponding model where the Higgs boson is represented by an elementary scalar field with the usual Yukawa- and self-interactions. In ref.[5] in case of one or two elementary Higgs doublets severe bounds based on vacuum stability between the heaviest Higgs boson and top-quark mass have been established. In fact these bounds are based on the requirement of the boundedness of the Hamiltonian. So far the natural question arises: are these bounds applicable in case of a composite Higgs boson, i.e. for the case of the auxiliary Higgs scalar field?

The aim of this note is to convince the reader that for the case of the composite Higgs boson the standard arguments based on the vacuum stability criterion are not applicable anymore and hence we cannot use the stability bound for the top-quark mass.

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mass. Therefore the discovery of the top-quark and Higgs boson in the prohibited region (from the point of view of the vacuum stability requirement) could mean the compositeness of the Higgs boson.

The paper is organized in the following way. In sect. 2 a brief derivation of the stability bound is given. In sect. 3 we give several examples which show that in case of the composite (auxiliary) Higgs boson the standard vacuum stability arguments are not applicable. Section 4 contains concluding remarks.

2. - The vacuum stability bound.

To be concrete let us consider the toy model with the Lagrangian

\[ \mathcal{L} = i \sum_{K=1}^{N} \bar{\psi}_K \gamma^\mu \gamma^5 \psi_K - h \sum_{K=1}^{N} \bar{\psi}_K \gamma^5 \varphi - \frac{\lambda \varphi^4}{12} + \frac{m^2 \varphi^2}{2} + \frac{1}{2} (\partial_\mu \varphi \partial^\mu \varphi). \]

Model (1) is renormalizable and it describes the interaction of N identical fermions with a scalar field. The Lagrangian of the model is invariant under the transformation

\[ \psi \rightarrow \gamma_5 \psi, \quad \varphi \rightarrow -\varphi. \]

The tree level potential of the model is

\[ V_0 = \frac{\lambda \varphi^4}{12} - \frac{m^2 \varphi^2}{2}. \]

For \( m^2 > 0 \) the equation

\[ \frac{\partial V_0}{\partial \varphi} = 0 \]

has a nontrivial vacuum solution

\[ \varphi_0 = \pm \sqrt{\frac{3m^2}{\lambda}} \]

which breaks the symmetry (2). After the symmetry breaking the fermions acquire a mass \( M_F = h \varphi_0 \) and the mass of the scalar boson is equal to \( m_H^2 = 2m^2 \). The one-loop correction to the renormalized effective potential (3) is

\[ V_1 = -\frac{4N}{64\pi^2} (h \varphi)^4 \ln \left( \frac{h \varphi^2}{h \varphi_0^2} \right) + \frac{1}{64\pi^2} (\varphi^2 - m^2)^2 \ln \left( \frac{\lambda \varphi^2 - m^2}{\lambda \varphi_0^2 - m^2} \right) + A \varphi^2 + B \varphi^4. \]

Here the constants \( A \) and \( B \) are determined from the renormalization conditions

\[ \left. \frac{\partial V_1}{\partial \varphi} \right|_{\varphi = \varphi_0} = 0, \quad \left. \frac{\partial^2 V_1}{\partial \varphi^2} \right|_{\varphi = \varphi_0} = 0. \]

For the effective potential \( V = V_0 + V_1 \) we require that the minimum \( \varphi = \varphi_0 \) has to be the deepest minimum, \( i.e. \) we require that \( V(\varphi) > V(\varphi_0) \) for all \( \varphi \). From this