Evidence for Pion-Pion Interactions from $s$-Wave Pion-Nucleon Scattering (*).

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Summary. — By fitting the $s$-wave partial amplitudes for $\pi \cdot N$-scattering, in an unphysical region, we have greatly extended the energy range over which we can examine the contribution from the process $\pi + \pi \rightarrow N' + N$. Our method shows up contributions from this process in states of isotopic spin $T=0$ and $T=1$. We can estimate the energies $t^\dagger$ for which this process is important. The form of these contributions is just what would be expected from considerations of angular momenta. Estimates of the amplitudes for $\pi + \pi \rightarrow N' + N$ are deduced. The $T=0$ amplitude is large, but in the $T=1$ case the amplitude is much smaller than the values which have been predicted from the nucleon isovector form factors.

1. - Introduction.

In another paper hereafter referred to as I, two of us (J.H. and T.D.S.) (1) gave an account of the evaluation of the dispersion relations for the $s$-wave partial amplitudes $f_0^{(T)}(s)$ in pion-nucleon scattering. ($T = \frac{1}{2}, \frac{3}{2}$ is the isospin and $s = [(M^2 + q^2)^2 + (\mu^2 + q^2)^2]^2$ is the energy variable. $M, \mu$, are the nucleon and pion masses and $q$ is the c.m. momentum). The relations are

\begin{equation}
\text{Re} f_0^{(T)}(s) = \text{(Born terms)}^{(T)} + \\
+ \frac{1}{\pi} \int_{(M+\mu)^2}^\infty \frac{\text{Im} f_0^{(T)}(s')}{s'-s} \text{ds'} + \frac{1}{\pi} \int_0^{(M-\mu)^2} \frac{\text{Im} f_0^{(T)}(s')}{s'-s} \text{ds'} + \Delta^{(T)}(s).
\end{equation}

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The second and third terms on the right are the physical integral (re-scattering) and the crossed integral; they come from the cuts \((M+\mu)^2 < s < \infty, 0 < s < (M-\mu)^2\) respectively (Fig. 1). From the known values of the phase shifts and the coupling constant \(f^2\), all terms in (1) can be evaluated except \(\Delta^{(\tau)}(s)\). This is the discrepancy, and gives the contribution to \(\text{Re} f^2_{\pi\pi}(s)\) from the cut \(+\infty < s < 0\) and the circle \(|s| = M^2 - \mu^2\) (Fig. 1); the cut \(-\infty < s < 0\) corresponds to effects whose range is \(\hbar/\mu c\) or less, and the circle arises from the process \(\pi^+ + \pi^+ \rightarrow N^0 + \overline{N}^0\). The right-hand portion of the circle can give comparatively long range effects in \(\pi^- N^+\) s-wave scattering. These should show up in an appreciable variation of \(\Delta^{(\tau)}(s)\) with \(s\) at low energies.

A sizable contribution from the right-hand portion of the circle implies an appreciable imaginary part of the amplitude for \(\pi^+ + \pi^+ \rightarrow N^0 + \overline{N}^0\), which in turn implies appreciable pion-pion scattering \((\pi^+ \pi^+ \rightarrow \pi^+ \pi^+)\) at fairly low energies (\(^2\)). It is convenient to use

\[
\Delta^{(+)} = \frac{1}{3}(\Delta^{(1)} + 2\Delta^{(2)}), \quad \Delta^{(-)} = \frac{1}{3}(\Delta^{(1)} - \Delta^{(2)}),
\]

\(\Delta^{(+)}\), \(\Delta^{(-)}\) correspond to the isospin values \(T = 0, 1\), respectively, for the \(\pi^+ + \pi^+ \rightarrow N^0 + \overline{N}^0\) system. FRAZER and FULCO (\(^3\)) have suggested that a resonance in the \(T = 1\) state of \(\pi^+ + \pi^+ \rightarrow \pi^+ + \pi^+\) at \(t^*_R = 4(\mu^2 + \bar{q}^2) \simeq 11.5\mu^2\) will explain the iso-

Fig. 2. - The discrepancies \(\Delta^{(1,2)}_{1,2}(s)\), in units \(\hbar = \mu = c = 1\). The subscripts 1, 2 refer to the values of \(\Delta^{(1,2)}(s)\) derived by using the sets of phase shifts \((a_1)\) and \((a_2)\) described in Section 2.
