On the Causal Propagation Function of a Dirac Field.

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Summary. — It is shown that the causal propagation function of a Dirac field can be split in two functions which satisfy a system of two integral equations similar to the couple of Low equations for the symmetrical charged fixed-source meson theory, in the one-meson approximation. These equations are solved and discussed.

1. — It has been shown by K. W. Ford (1) that the Fourier transform of the causal propagation function of a boson field, interacting relativistically in a given, but unspecified way with other fields, satisfies an integral equation which is formally identical with the Low equation for a neutral fixed-source meson theory, in the one-meson approximation. The equation contains in place of the known cut-off function, an unknown function, fixed in principle by the interaction. Ford shows that a condition for the existence of the solution of this equation leads to the limitation on the asymptotic behaviour of the "vertex function" previously found by other methods by Lehmann, Symanzik and Zimmermann (2).

In this paper we shall attempt to show that an analogous situation occurs also with the causal propagation function of a Dirac field, in the sense that, by means of a convenient decomposition of it suggested by relativistic invariance requirements, two functions can be defined, which satisfy a system of two integral equations, which is formally similar to the couple of Low equations

for the symmetrical charged fixed-source meson theory, in the one-meson approximation. In this case the integral equations contain two unknown functions which have the role of a natural cut-off built in in the theory. Also in this case the integrability conditions of this system lead, if the two unknown functions are approximated by limiting the intermediate states, and assuming a pseudoscalar interaction of the Fermi field with a pseudoscalar neutral boson field, to a condition for the asymptotic behaviour of the «vertex function» for final fermion and boson on the energy shell (2).

2. - The starting point of our discussion is a treatment of the \( S_{\alpha\beta}^{(+)} \) function

\[-iS_{\alpha\beta}^{(+)}(x - x') = \langle \Omega \Psi_{\alpha}(x) \overline{\Psi}_{\beta}(x') \Omega \rangle \]

(where \( \Psi_{\alpha}, \overline{\Psi}_{\beta} \) are the field operators and the brackets indicate an average value in the vacuum state \( \Omega \)) analogous to the one followed in ref. (1) in the case of the \( A^{(+)} \) function. We put:

\[ \langle \Omega \Psi_{\alpha}(x) \overline{\Psi}_{\beta}(x') \Omega \rangle = \sum_{\nu} \langle \Omega, \Psi_{\alpha}(x) \Phi_{\nu}^{in}, \overline{\Psi}_{\beta}(x') \Omega \rangle \]

introducing into the sum a complete set of physical intermediate states, belonging to the same value of baryonic quantum number. The label \( \nu \) indicates the number and the type of the particles present in the state.

It is easily seen that this expression can be written in the form

\[ \langle \Omega \Psi_{\alpha}(x) \overline{\Psi}_{\beta}(x') \Omega \rangle = \left[ \int S_{\nu}^{'\prime}(x - \xi) F(\xi - \eta) \tilde{S}_{\nu}^{'\prime}(\eta - x') d\xi d\eta \right]_{\alpha\beta}, \]

where \( S_{\nu}^{'\prime}, \tilde{S}_{\nu}^{'\prime} \) are the causal propagation function defined by

\[ S_{\nu\alpha\beta}^{'\prime}(x - y) = \langle \Omega, T(\Psi_{\alpha}(x) \overline{\Psi}_{\beta}(y)) \Omega \rangle \quad \tilde{S}_{\nu\alpha\beta}^{'\prime}(x - y) = \langle \Omega, \overline{T}(\Psi_{\alpha}(x) \overline{\Psi}_{\beta}(y)) \Omega \rangle. \]

\( T \) and \( \overline{T} \) indicate invariant chronologically and antichronologically ordered products respectively.

The function \( F \) can be thought of as a sum of the contributions of the intermediate states with a given number \( n \) of present particles. It is still a \( 4 \times 4 \) matrix.

We now define

\[ F(x) = \sum_{n \geq n} F_n(x). \]

(3) In ref. (2) this limitation has been stated without proof.