Pion-Hyperon Resonance from $\bar{K}$ Nucleon Coulomb
Constructive $S$-Wave Parameters.

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The elastic $K-N$ and $\bar{K}-N$ scattering amplitudes exhibit a relevant long range interaction due to the possibility of exchanging pions between the $K$-mesons and the nucleons (1). To be more specific, by assuming double dispersion relations, it has been shown that the low angular momentum phase-shifts may be strongly affected by the presence of the dynamical branch cut related with the pion exchanges (2).

Because of the production of $\Lambda$- and $\Sigma$-hyperons connected with the $\bar{K}-N$ interaction, it is interesting to see whether a $\bar{K}-N$ scattering amplitude properly continued below the $\bar{K}-N$ physical threshold might provide information about the $\pi$-hyperon systems. DALITZ and TUAN (3) starting from a zero range solution do find a resonance in the $I=1$ $\pi$-hyperon $S$-wave amplitude, in agreement with the experimental data. Their scattering solution which behaves in this way has the peculiar character of producing destructive interference with the $K^+p$ Coulomb force (4).

(4) We remark that this solution does not explain the energy behaviour of the $\Sigma^-/\Sigma^+$ production rate suggested by the present data. See: R. H. DALITZ and S. F. TUAN: Ann. of Phys., 10, 307 (1960).
Experimental evidence, although not definite on this point, seems to favour a constructive interference (2).

In this note we like to emphasize that a Coulomb constructive K-π scattering amplitude can generate a resonance in the I=1 S-wave amplitude of the pion-hyperon system, if the long range force (due to the KK-ππ interaction) is taken into account.

In previous works the low energy behaviour of the K-Λ and K-Σ̅ scattering has been interpreted in terms of a simple model in which a strong pion-pion interaction takes place in the J=1, I=0 state. It has been found that a sharp resonance at \( t_R \approx 12 \) (\( t = \) square of the total c.m. energy in pion mass units) gives results consistent with the present experimental data (5,7).

Recently, different pion production experiments (8) as well as theoretical analyses of the electromagnetic form factors of the nucleons (9) seem to be in favour of a resonant J=1, I=0 three-pion bound state at \( t_R \approx 5 \) (9,10).

We are therefore led to consider two possible sources of the long range interaction. It is fortunate that their contributions are additive in the I=1 K-Λ and K-Σ̅ systems (11), giving rise to long range forces similar to those obtained in ref. (12). This still allows an approximate description of the effects of the pion interactions along the dynamical branch cut in terms of a single pole, which would replace the separate contributions of the two- and three-pion states.

We point out that the two-pion forces are strongly repulsive in the I=1 K-Λ state, providing us with a simple interpretation of the low energy behaviour of the K+̅ scattering cross-section.

The amplitude for Σ̅-hyperon scattering is related to the K-̅-proton amplitude through the unitarity condition (12,13). In order to decide about the existence of a bound I=1 state, in the absence of a rigorous analytical continuation in the K-Σ̅ elastic amplitude which was approximated in eq. (5) of ref. (5), we have to make sure that it reproduces dynamically as a resonance in the pion-hyperon elastic scattering.

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(7) The additivity of the contributions due to the two- and three-pion systems to the long range interactions can be argued from Sect. 5 of ref. (5).
(8) The long range parts of the K-Λ and K-Σ̅ potentials due to pion exchanges are simply related to each other, as long as the reactive effects of the absorption processes in the K-Λ system are not too strong in this region. This should be the case, since the absorption interactions take place at a shorter range, corresponding to the exchange of at least one pion and K-meson.
(9) We notice that the K-Λ partial waves scattering amplitudes exhibit a peculiar superposition of the unitarity cut (in the s plane) with the branch cut due to the pion exchanges. The latter continues below the former up to the value \( s = \left( \sqrt{m_\Lambda^2 + m_π^2} + \sqrt{m_π^2 + m_π^2} \right)^2 \). It can be easily shown that this overlapping does not forbid the possibility of writing dispersion relations for the partial waves, as the Mandelstam representation gives a unique prescription for separating the overlapping singularities. See also: S. MANDELSTAM: *Phys. Rev. Lett.*, 4, 84 (1960).