Analytic Properties of Bound States in Potential Theory.

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Summary. — The analytic properties of the vertex functions of a composite particle (« deuteron ») are analyzed in a potential model. The results are compared with the « anomalous » singularities of the relativistic perturbation theory.

1. — Introduction.

One of the most interesting programs in dispersion theory is the application of the dispersion methods to the study of problems of nuclear physics.

The advantage of the dispersion approach on the conventional ones, lies in the fact that it allows to take into account in an appropriate manner the relativistic effects.

The main difficulty in such an application is due to the fact that the nuclear systems are loosely bound and relativistic perturbation theory predicts a location of the branch points, which is rather different from what one would guess from unitarity (1).

Such « anomalous » thresholds are certainly the main feature of the dispersion relations for nuclear problems, and therefore, a deeper understanding of their meaning is needed for a successful development of the dispersion program.

At present it has not been possible to go beyond lowest order perturbation theory and even understand completely the meaning of the use of Feynman

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graphs for nuclear problems, since we do not have any hamiltonian theory from which such graphs can be derived.

We think, therefore, that a comparison between the results obtained in relativistic perturbation theory and in the conventional approach, is very instructive. In this paper, we will show that also in the case of potential theory the vertex functions in which one or two deuteron lines enter can be represented in a spectral form.

We shall assume that the deuteron is an eigenstate of a Schrödinger equation with a potential of the form

\[ V(r) = \int_{\mu_0}^{\infty} g'(\mu) \frac{\exp[-\mu r]}{r} d\mu. \]

We know that for potentials of the type (1.1) it has been possible to derive a Mandelstam representation for nucleon-nucleon scattering (2).

The method used in order to derive the spectral representation is based on a Laplace expansion of the deuteron wave function; such a method has been used for scattering problem by Martin (3). A corresponding form of the wave function in momentum space has been introduced by Blakenbecler and Cook (4).

For the vertex functions considered the potential method fully confirms the predictions of lowest order perturbation theory, and allows to obtain the analytic properties of all orders in perturbation theory, when a static approximation is taken.

2. - The method.

2'1. - Let us consider a system of 2 spinless particles (which we shall call «proton» and «neutron»), interacting through a central potential \( V(r) \)

\[ V(r) = \int_{\mu_0}^{\infty} g'(\mu) \frac{\exp[-\mu r]}{r} d\mu = \int_{\mu_0}^{\infty} g(\mu) \exp[-\mu r] d\mu, \]

(2.1)

\[ g(\mu) = \int_{\mu_0}^{\infty} g'(\lambda) \theta(\mu - \lambda) d\lambda. \]


(3) A. Martin: Nuovo Cimento, 14, 403 (1959); 15, 99 (1960).

(4) L. F. Cook and R. Blakenbecler: to be published.