CORRELATION-SPECTRAL TRANSFORMATION OF SIGNALS IN ADAPTIVE SEQUENTIAL BASIS

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A procedure for deriving a system of stochastic sequential basis functions is described. It is shown that the use of these functions for correlation and spectral analysis of stochastic signals substantially reduces and simplifies computing operations. A simple method of calculation of the spectrum of variances is given.

Keywords: correlation-spectral transformation, adaptive sequential basis, discrete Fourier transformation.

In solving problems involving processing of random processes, the methods of spectral and correlation analysis are most frequently used. The classical methods of digital correlation processing and spectral discrete Fourier transformation (DFT) call for application of labor-consuming calculations to which the multiplication belongs. This essentially affects the rate of the analysis which is an important measure of efficiency of the method. Therefore reduction of the number and/or simplification of analytical operations is a vital problem.

The widespread methods of fast Fourier transformation (FFT) use less operations of multiplication than DFT, thus accelerating processing by one or two orders of magnitude [1]. The sequential spectral methods [2] are even faster; they use an insignificant number of operations, and multiplications are excluded as a result of the use of discontinuous bases, for example, of the system of Walsh functions.

Fast methods of correlation treatment include, first of all, those excluding or simplifying multiplication (the Stieltjes method, relay, and sign methods), which is achieved by robust amplitude quantization (even to clipping, i.e., to a limiting two-sided restriction of amplitude) of one or both of the signals [3].

We will consider below the methods in which a stochastic sequential basis is used for spectral-correlation transformations [4]. These methods simplify calculations and hardware and do not affect the accuracy of the analysis.

STOCHASTIC BASIS

Let \( x(t) \) be a differentiable repeatedly stochastic process with zero (for the sake of convenience in explanation) mathematical expectation. Then an adaptive sequential basis exists on the interval \( 0 < t < T \) for an arbitrarily large \( T \). Let us show that processing of a signal in this basis allows us to obtain the amplitude-frequency spectrum \( x(t) \).

Let \( \theta(t) \) be the unit step function:

\[
\theta(t) = \begin{cases} 
1 & \text{for } t \geq t_i, \\
0 & \text{for } t < t_i,
\end{cases}
\]

where \( t_i \) is the moment of occurrence of the \( i \)th extremum in the signal.

Denote by \( J(\Delta t_c) \) a rectangular unit impulse of duration \( \Delta t_c = |t_j - t_i| \). Let us determine

\[
J(\Delta t_c) = [\theta(t - t_j) - \theta(t - t_i)](-1)^d,
\]
\[ d \begin{cases} 0 & \text{for } [x(t_j) - x(t_i)] > 0, \\ 1 & \text{for } [x(t_j) - x(t_i)] < 0, \end{cases} \]

where \( x(t_i), x(t_j) \) are amplitude values of the signal at the moments \( t_i \) and \( t_j \) of occurrence of the extrema.

Then \( \int_0^T x'(t)J(\Delta t_c)d\tau \) will separate from the process \( x(t) \) a section enclosed by zero values of the derivative at the instants \( t = t_i \) and \( t = t_j \), and will also determine the amplitude of the oscillatory half-wave of duration \( \Delta t_c \):

\[ \int_0^T x'(t)J(\Delta t_c)d\tau = |x(t_j) - x(t_i)|. \]

We will consider the basis function \( G_c(t) \) as a collection of positive and negative pulses of unit amplitude, whose duration is determined by the interval \( \Delta t_c \) between extrema being analyzed on the time interval \( 0 < t < T \):

\[ G_c(t) = \sum J(\Delta t_c). \]  

(1)

To derive a basis system of functions, we will apply the following rule:

— each function can be generated after measuring time intervals and amplitude increments (with regard for their sign) between neighboring extrema, with the sequence of rectangular pulses being synthesized based on their values;

— the first basis function is formed from the initial array of extrema, the subsequent ones from the filtered extrema of the previous array (the filtration is carried out by eliminating minima or maxima, i.e., even or odd discretes), and among the remaining ones we find new extrema, determine new amplitude and time increments, and form rectangles;

— the obtained sequences of pulses are clipped, thus reducing them to the unit amplitude.

Hence, the basis system is formed from the set of functions \( \{G_c(t)\} \) where durations of rectangular pulses \( \{\Delta t_c\} \) with unit amplitude reflect both distance between the next extremum points in the previous array (including the initial one) and distance between extremum points in envelopes of these extrema. By an envelope of extrema we mean here a curve approximating a discrete sequence of maxima or of minima of the previous array of extrema.

It is easy to verify that such system of functions is orthonormalized, i.e.,

\[ (G_i, G_j) = \begin{cases} 0 & \text{for } i \neq j, \\ 1 & \text{for } i = j, \end{cases} \]

and possesses the group properties with respect to the operation of addition:

\[ G_i(t) + G_j(t) = G_m(t); \ G_m(t) \in \{G_c(t)\}. \]

(2)

Thus, the system of functions \( \{G_c(t)\} \) is a stochastic sequential system whose adaptive character is due to the algorithm of discretization of the signal being studied. Note that the study of completeness is not applied in sequential basis systems [2].

Let us expand the derivative of the process \( x'(t) \) into a series of functions \( \{G_c(t)\} \):

\[ x'(t) = \sum_{i=1}^T a_i(\Delta t_i)(t), \]

(3)

where

\[ a_i(\Delta t_i) = \frac{1}{T} \int_0^T x'(t)G_i(t)dt = \frac{1}{Q} \sum_{n,m} |x(t_n^i) - x(t_m^i)|. \]

(4)

Here, \( Q \) is the number of pulses of duration \( \Delta t_i \) falling on the interval \( 0 < t < T \); \( x(t_n^i), x(t_m^i) \) are the extremum points of the process \( x(t) \), with the distance between them being equal to \( \Delta t_i \).

Integrating (3), we obtain

\[ x(t) = \int_0^T \sum_{i=1}^T a_i(\Delta t_i)G_i(t)dt + \varepsilon(t). \]

(5)

It is shown in [4] that the coefficients \( \{a_i\} \) determined by formula (4) minimize the root-mean-square error \( \varepsilon(t) \).