Solutions of Einstein's Field Equations I and II (Singular Cases).

KAUSHIK CHANDRA SHEKHAR
Department of Mathematics, Asfa Wonen School - Addis Ababa

(recevuto il 18 Dicembre 1961)

Summary. Prof. Hlavaty has given (1) the solution of Einstein's first set of field equations for the first and the second classes of the space-time in the only physically admissible case for index of inertia two. The object of this paper is to consider the solution for these two classes in the singular cases for all possible indices of inertia.

1. Introduction.

In the final attempt for the generalization of the general Theory of Relativity or the Theory of Gravitation the four-dimensional space $\chi_4$ is referred to all real co-ordinate systems but accepting the only co-ordinate transformations for which

\[ A \in \text{det} \begin{pmatrix} \hat{\gamma}^{\rho}_{\mu} \\ \hat{\gamma}^{\mu}_{\rho} \end{pmatrix} \neq 0. \]

And it is endowed with a real quadratic non-symmetric tensor $g_{\lambda\mu}$ breakable in to its symmetric and skew-symmetric parts:

\[ g_{\lambda\mu} = h_{\lambda\mu} + k_{\lambda\mu}, \]

i.e.

\[ h_{\lambda\mu} \text{det} g_{\lambda\mu} = \frac{1}{2} (g_{\lambda\mu} + g_{\mu\lambda}); \]

\[ k_{\lambda\mu} \text{det} g_{\lambda\mu} = \frac{1}{2} (g_{\lambda\mu} - g_{\mu\lambda}). \]

(1) V. Hlavaty: Geometry of Einstein's Unified Field Theory (Groningen, 1958).
Since \( k_{j\mu} \) is the skew-symmetric part of the tensor \( g_{\lambda\mu} \), the indices can be lowered or raised by means of the symmetric tensor \( h_{j\mu} \) and its tensor-inverse \( h^{\lambda\mu} \). But the lowering or raising of two indices simultaneously is performed by the help of covariant or contravariant indicators defined as follows:

The contravariant indicator \( \varepsilon_{\alpha\beta\gamma\delta} \) is a tensor-density of weight \(-1\) skew-symmetric in all its indices whose components have in all coordinate systems the following numerical values.

\[ a) \quad +1, \text{ whenever } \omega_{\mu\lambda\nu} \text{ is an even permutation of } 1, 2, 3, 4. \]
\[ b) \quad -1, \text{ whenever } \omega_{\mu\lambda\nu} \text{ is an odd permutation of } 1, 2, 3, 4. \]
\[ c) \quad 0, \text{ in all the remaining cases.} \]

Denoting by \( g, h \) and \( f \) the determinant of \( g_{\lambda\mu}, h_{j\mu} \) and \( k_{j\mu} \) respectively, let us put the scalars

\[ g \overset{\text{def}}{=} \frac{1}{h}, \]
\[ k \overset{\text{def}}{=} \frac{f}{h}, \]
and the tensors

\[ k_0 \overset{\text{def}}{=} k^\nu(= k_{\lambda\mu} h^{\lambda\nu}); \quad (p) k_2 \overset{\text{def}}{=} (p-1) k_{\alpha} k_{\alpha}^\nu, \quad p = 2, 3, 4, \ldots \]

so that the scalar

\[ 4K \overset{\text{def}}{=} k_{\alpha\beta} k_{\alpha\beta} = k_{\alpha\beta} k_{\beta\gamma} h^{\gamma\alpha} h^{\beta\gamma} = -(p) k_{\alpha}^\nu. \]

And then

\[ g = 1 + 2K + k. \]

The space-time and hence the tensor \( g_{\lambda\mu} \) or \( k_{j\mu} \) is said to be of the

\[ a) \quad \text{First class if } k \neq 0. \]
\[ b) \quad \text{Second class if } k = 0, \quad K \neq 0. \]
\[ c) \quad \text{Third class if } k = 0, \quad K = 0 \quad \text{but } (p) k_2^\nu \neq 0. \]