Solutions of Einstein's Field Equations I and II (Singular Cases).

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Summary. Prof. HLAVATY has given (1) the solution of Einstein's first set of field equations for the first and the second classes of the space-time in the only physically admissible case for index of inertia two. The object of this paper is to consider the solution for these two classes in the singular cases for all possible indices of inertia.

1. - Introduction.

In the final attempt for the generalization of the general Theory of Relativity or the Theory of Gravitation the four-dimensional space $\chi_4$ is referred to all real co-ordinate systems but accepting the only co-ordinate transformations for which

\begin{equation}
A \equiv \det \left( \frac{\partial x^\nu}{\partial x^{\nu'}} \right) \neq 0.
\end{equation}

And it is endowed with a real quadratic non-symmetric tensor $g_{\lambda\mu}$ breakable in to its symmetric and skew-symmetric parts:

\begin{equation}
g_{\lambda\mu} = h_{\lambda\mu} + k_{\lambda\mu},
\end{equation}

i.e.

\begin{equation}
h_{\lambda\mu} \equiv \det g_{\lambda\mu} = \frac{1}{2} (g_{\lambda\mu} + g_{\mu\lambda}) ;
\end{equation}

\begin{equation}
k_{\lambda\mu} \equiv \det g_{\lambda\mu} = \frac{1}{2} (g_{\lambda\mu} - g_{\mu\lambda}) .
\end{equation}

(1) V. HLAVATY: Geometry of Einstein's Unified Field Theory (Groningen, 1958).
Since $k_{\lambda \mu}$ is the skew-symmetric part of the tensor $g_{\lambda \mu}$, the indices can be lowered or raised by means of the symmetric tensor $h_{\lambda \mu}$ and its tensor-inverse $h^{\lambda \mu}$. But the lowering or raising of two indices simultaneously is performed by the help of covariant or contravariant indicators defined as follows:

The contravariant indicator \( \epsilon_{\alpha \mu \lambda \nu} \) is a tensor-density of weight \( -1 \) skew-symmetric in all its indices whose components have in all coordinate systems the following numerical values.

- a) \( +1 \), whenever \( \omega_{\mu \lambda \nu} \) is an even permutation of \( 1, 2, 3, 4 \).
- b) \( -1 \), whenever \( \omega_{\mu \lambda \nu} \) is an odd permutation of \( 1, 2, 3, 4 \).
- c) \( 0 \), in all the remaining cases.

Denoting by \( g, h \) and \( f \) the determinant of \( g_{\lambda \mu}, h_{\lambda \mu} \) and \( k_{\lambda \mu} \) respectively, let us put the scalars

\[(1.3a) \quad g \overset{\text{def}}{=} \frac{1}{h} ,\]
\[(1.3b) \quad k \overset{\text{def}}{=} \frac{f}{h} ,\]
and the tensors

\[(1.3c) \quad k_1^\alpha \overset{\text{def}}{=} k_1^\alpha (= k_{\lambda \mu} h^{\lambda \nu}) ; \quad (p) k_2^\alpha \overset{\text{def}}{=} (p-1) k_2^\alpha k_2^\nu , \quad p = 2, 3, 4, ...\]

so that the scalar

\[(1.3d) \quad 4K \overset{\text{def}}{=} k_{\alpha \beta} k^{\alpha \beta} = k_{\alpha \beta} k_{\gamma \delta} h^{\gamma \alpha} h^{\delta \beta} = -(p) k_2^\alpha .\]

And then

\[(1.4) \quad g = 1 + 2K + k .\]

The space-time and hence the tensor \( g_{\lambda \mu} \) or \( k_{\lambda \mu} \) is said to be of the

- a) First class if
\[(1.5a) \quad k \neq 0 .\]
- b) Second class if
\[(1.5b) \quad k = 0 , \quad K \neq 0 .\]
- c) Third class if
\[(1.5c) \quad k = 0 , \quad K = 0 \quad \text{but} \quad (3) k_2^\alpha \neq 0 .\]