On the Interpretation of the \( \tau \)-Functions in Heisenberg's Nonlinear Theory of Elementary Particles.

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Summary. — In this note, some particular cases of a general formula derived in a preceding paper are investigated. The structure of the results seems to confirm a conjecture, which was put forward by Heisenberg some time ago. A possible way to get a more complete interpretation of our formulae is also suggested.

In a previous paper (1) we have derived a spectral formula for the function:

\[
\tau_0(xy|zw) = \langle T_{\sigma_i(x) \sigma_j(y) \sigma_k(z) \sigma_l(w)} \rangle_0,
\]

where the two-component field operator \( \sigma_i(x) \) satisfies the differential equation:

\[
- \frac{\partial^2}{\partial x^2} \sigma_i(x) = \mu^2 \sigma_\mu \sigma_{\mu} \sigma_i(x) \sigma_j(x) \sigma_k(x) \sigma_l(x).
\]

This was done as a preliminary step of a general investigation of the structure of the \( n \)-point function in Heisenberg's non linear spinor theory, written in the formalism of Gürsey's wave matrix according to Dürr. The spectral

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representation we are speaking of can be written as:

\[ \tau_{\sigma}(x_{ij} y_{ik}) = \delta_{\alpha} \delta_{\beta,\gamma} \int (dm) \prod_{p<s} \Lambda^{p}(y_{p}; m_{p}) M(1 \ldots 6) + \]

\[ + \sum_{i=1}^{3} \int (dm) M_{i}^{p}(1 \ldots 6) \prod_{p<s} \Lambda^{p}(y_{p}; m_{p}) E^{(\sigma^{u})}_{i;\lambda}(y; m) + \]

\[ + \sum_{i=1}^{3} \int (dm) N_{i}^{p}(1 \ldots 6) \prod_{p<s} \Lambda^{p}(y_{p}; m_{p}) E^{(\sigma^{u})}_{i;\lambda}(y; m) + \]

\[ + \sum_{i=1}^{3} \int (dm) N_{i}^{p}(1 \ldots 6) \prod_{p<s} \Lambda^{p}(y_{p}; m_{p}) E^{(\sigma^{u})}_{i;\lambda}(y; m) , \]

where \((\sigma_{1,2,3}) = (16, 25, 34), (\sigma_{1,2,3}) = (61, 52, 43)\) and, for example, \(E_{i;\lambda}(y; m) = K_{i}(y_{1}; m_{1}) K^{p}_{i}(y_{4}; m_{4})\) and so on. The spectral functions \(M_{i}, N_{i}(i=1, 2, 3)\) are connected by various relations, which have the effect of reducing them to only four independent functions; moreover, they satisfy various symmetry requirements and, together with \(M(1 \ldots 6)\), certain integral conditions, to avoid undesired \(\delta\)-like singularities that arise when two or more points \(x_{i}\) coincide and that, on the other hand, cannot appear if one wants to establish, in a correct fashion, the connection of (3) with the two-point function, by means of the differential eq. (2).

These conditions will now be used to derive, from the general expression (3), formulae relating to the following particular cases:

\((4a)\)  \(\tau_{\sigma}(x_{ij} y_{ik})\),

\((4b)\)  \(\tau_{\sigma}(x_{ij} y_{ik})\).

In the first case, putting:

\[(5) \quad P^{(1)}(1 \ldots 4) = \left(- \frac{i}{8\pi^{2}} \right)^{2} \int d(\lambda^{2}) d(\mu^{2}) \lambda^{2} \mu^{2} \log \lambda \log \mu M(\lambda^{2} 1234 \mu^{2}) . \]

\[(6) \quad Q^{(1)}(1 \ldots 4) = \left(- \frac{i}{8\pi^{2}} \right)^{2} \int d(\lambda^{2}) d(\mu^{2}) \lambda^{2} \mu^{2} \log \lambda \log \mu \cdot \]

\[\cdot \{M_{i}^{(1)}(\lambda^{2} 1342 \mu^{2}) + M_{i}^{(1)}(\lambda^{2} 3124 \mu^{2}) + M_{i}^{(1)}(\lambda^{2} 2341 \mu^{2}) + M_{i}^{(1)}(\lambda^{2} 3214 \mu^{2})\} , \]

\[(7) \quad R^{(1)}(1 \ldots 4) = \left(- \frac{i}{8\pi^{2}} \right)^{2} \int d(\lambda^{2}) d(\mu^{2}) \lambda^{2} \mu^{2} \log \lambda \log \mu \cdot \]

\[\cdot \{N_{i}^{(1)}(\lambda^{2} 1342 \mu^{2}) + N_{i}^{(1)}(\lambda^{2} 3124 \mu^{2}) + N_{i}^{(1)}(\lambda^{2} 2341 \mu^{2}) + N_{i}^{(1)}(\lambda^{2} 3214 \mu^{2})\} . \]