On the Equivalence 
between Pseudo-Scalar and Pseudo-Vector Couplings.

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Summary. — A proof is given of the equivalence between the pseudo-scalar coupling theory, and a pseudo-vector coupling theory in which the propagator of the fermion field is simply \((p^2 - m^2)^{-1}\). In the latter theory only two couplings are present. One is the usual pseudo-vector interaction and the other is the familiar second order term containing the squares of both the fermion and the boson fields. No other kind of interaction (or nonlinearity) is necessary for the equivalence to hold in any order of the perturbation expansion.

1. — Introduction.

From a field-theoretical point of view the pseudo-scalar coupling theory (being renormalizable) is the most important way of describing the interaction between pions and nucleons. Nevertheless, it has some times been found convenient to represent this theory in such a form that the above mentioned interaction is eliminated, by means of a unitary transformation, in favour of the pseudo-vector coupling \(^{(1,2)}\), which is more physical for the nonrelativistic limit \(^{(2,3)}\). That goal is achieved not without obscuring the physical picture because that procedure invariably leads to complicated nonlinear interaction terms that render almost useless all but the lowest orders of the so-obtained pseudo-vector theory.

It is our purpose to show that the equivalence holds exactly and in a simple form, when one considers not the «normal» pseudo-vector theory which implies essential nonlocality \(^{(4)}\) but another kind of theory having as a distinctive feature the fact that the free spinor field is only supposed to obey the Klein-Gordon equation of motion. In other words, the propagator for the fermion is taken to be \((p^2 + m^2)^{-1}\) instead of the «normal» \((i\gamma \cdot p + m)^{-1}\). This is in line with our previous work for the case of the interaction with the electromagnetic field \(^{(5)}\) and we intend to discuss elsewhere the reasons behind the adoption of such a method and its relation to the minimal interactions.

2. – Description of the theory.

We shall suppose that all the familiar steps (starting from a Lagrangian) necessary to derive the usual perturbation treatment of the scattering matrix has been carried out. However, one difference must be noticed, the free (or interaction-picture) fields are here supposed to obey only the Klein-Gordon equation of motion. Dirac’s equation is considered as an initial condition and the interaction is to be chosen in such a way that any final fermion will automatically satisfy the Dirac equation if so do the initial fermions.

The condition imposed on the interaction is a necessity if one wants to avoid inconsistencies and it is particularly needed to assure the unitarity of the procedure. In fact, the usual propagator

\[
\frac{1}{i\gamma \cdot p + m} = \frac{m - i\gamma \cdot p}{p^2 + m^2},
\]

has a numerator which is proportional to a projection operator that allows only the actual particles to contribute to any real process, irrespectively of the type of coupling. It this projection operator is suppressed its role must be played by the interaction.

We shall see in what follows that this requirement is satisfied when one considers, together with the usual pseudo-vector term, another second order coupling term containing the squares of both, the fermion and the boson fields.

In order to simplify the presentation of the theory we will restrict ourselves to the consideration of the neutral case \(^{(6)}\), leaving aside all the non-essential kinematical factors.

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\(^{(6)}\) See however Appendix II.