The Relativistic Limit of the Theory of Vector Mesons.

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Summary. — The analogue of the Cini-Touschek transformation (relativistic limit) in the case of vector mesons is considered. The spinor form of Maxwell equations is obtained after this transformation is performed.

Introduction.

The nonrelativistic limit of the Dirac, Klein-Gordon and Proca equation is particularly exhibited after a Foldy-Wouthuysen transformation is performed. This leads, in fact, to a representation of the wave equation in which the operators representing physical quantities can be easily defined and directly related to the corresponding ones of the Schrödinger-Pauli theory.

The corresponding problem for the discussion of the ultra-relativistic limit in which the mass of the particle may be neglected with respect to its kinetic energy, has also been investigated. Cini and Touschek (1) have shown that the Dirac equation, for nonzero mass particles, may be put into the form of the neutrino equation by means of a unitary transformation. The ultra-relativistic limit of the theory of spin-\(\frac{1}{2}\) particles is therefore exhibited by the Cini-Touschek representation of the Dirac equation.

The aim of the present work is to investigate the ultra-relativistic limit of the theory of vector mesons. In such a way the spinor form of the photon wave equation as given by Oppenheimer (2), Good (3) et al. will be obtained.

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If the vector meson wave field is quantized the above transformation leads to the well-known commutation relations between the electric and magnetic fields.

The Sakata-Taketani (4) formulation of the wave mechanics of vector mesons is used here. According to that the second-order equation is replaced by a Schrödinger-like equation of the first order in the time derivative:

\[ i\hbar \frac{\partial \Psi}{\partial t} = H\Psi, \]

where

\[ H = i\sigma_1 \left( \frac{p^2}{2m} - \frac{(S\cdot p)^2}{m} \right) + \sigma_2 \left( mc^2 + \frac{p^2}{2m} \right), \]

\( \sigma_1, \sigma_2, \sigma_3 \) are the Pauli matrices and \( S_1, S_2, S_3 \) are the components of the spin-one operator. The scalar product of two wave functions \( \Psi_1 \) and \( \Psi_2 \) is defined by

\[ (\Psi_1, \Psi_2) = \int \Psi_1^* \Psi_2 \, dx. \]

For brevity's sake the notation

\[ O_1 = i \frac{p^2}{2m} - i \frac{(S\cdot p)^2}{m}, \quad O_2 = mc^2 + \frac{p^2}{2m}, \]

will be used throughout.

1. The Cini-Touschek transformation.

We consider an Hamiltonian of the type

\[ H = \sigma_1 O_1 + \sigma_2 O_2, \]

where \( O_1 \) and \( O_2 \) are commuting operators, and we assume that a matrix \( U \) exists, such that

\[ UH\bar{U} = \sigma_2 H_2. \]