Operatorial Expansion of the Product of Two Currents.

L. Bonora and I. Vendramin
Istituto di Fisica dell'Università - Padova
(ricevuto il 1º Luglio 1970)

Summary. — On the grounds of general principles and a certain set of hypotheses we build up the most general expansion of the product of two $U_3 \otimes U_3$ currents in neighbouring space-time points, following the Wilson method. By means of this expansion we determine a certain number of current-current and current-derivative-of-current equal-time commutators. These are in agreement with the results obtained by other methods. As an application, we have shown that the structure of stress-energy tensor suggested by Sugawara can be embodied as a limiting case in Wilson's theory.

1. — Introduction.

Recently equal-time commutators (ETCR's) of currents were obtained (1-10) by means of broken-scale-invariance considerations (24) and through relations among Schwinger terms, implied by covariance arguments (5). The usefulness

of these deductions lies in particular in the possibility of obtaining predictions about asymptotic electroproduction and neutrino-production cross-sections (6,7) and electromagnetic-mass differences (8). With the hypotheses of ref. (1) and ref. (10), we show here that one obtains essentially the same results by a different method, that is starting from the most general form of expansion of the product of two currents at nearby space-time points, following Wilson's theory (9).

Our second aim is to show that Sugawara theory on the structure of the stress-energy tensor as a quadratic function of currents, appears as a particular case of Wilson's theory, provided operator products at the same point are interpreted as limits for vanishing separation of products of the same operators at neighbouring points.

2. - The set of hypotheses.

It is useful to summarize the hypotheses underlying our consideration. We assume the validity of Wilson method (9) and consequently the related assumptions:

a) there exist operator product expansions for products of two local operators near the same space-time point;

b) strong interactions become scale invariant at short distances. The scaling laws of strongly interacting fields may differ from the ones of free fields because of renormalization effects.

Besides we assume (1-10):

c) the only local operators with dimension $d<4$ in the Wilson expansion are the following:

- $c_1$ the unit operator $I$ with $d = 0$;
- $c_2$ the currents $J^a_\mu(x)$ ($a = 1, \ldots, 8$ for vector currents; $a = 1, \ldots, 8$ for axial vector currents). They have $d = 3$ and belong to $[(1, 0, -1), (0, 0, 0)] \oplus [(0, 0, 0), (1, 0, -1)]$ representation of $U_3 \otimes U_3$;
- $c_3$ the derivatives of these currents, $\partial_\mu J^a_\nu(x)$ with $d = 4$;
- $c_4$ the stress-energy tensor $\theta_{\mu\nu}(x)$ with $d = 4$;
- $c_5$ the scalar and pseudoscalar densities $w(x)$ with $d = \Lambda(1 < \Lambda < 4)$. They belong to $[(1 + s, s, s), (-s, -s, -1 - s)] \oplus [(-s, -s, -1 - s), (1 + s, s, s)]$ representation of $U_3 \otimes U_3$, with $s$ integer;
- $c_6$ the scalar and pseudoscalar densities $w_s(x)$ with $d = \Lambda'(1 < \Lambda' < 4)$. They belong to $[(s' + s', s', s'), (-s' - s' - s')] \oplus [(-s' - s' - s'), (s', s', s')]$ representation of $U_3 \otimes U_3$, with $s' \neq 0$ integer;
- $c_7$ possible derivatives of the densities $c_5$ and $c_6$ of $d<4$;