Integral Representations of the Wave and Transition Operators in Nonrelativistic Scattering Theory.

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Summary. — Hilbert-space versions of the Lippmann-Schwinger equations for wave operators and the usual T-matrix formulae are derived by using the theory of Riemann-Stieltjes integrals of operator-valued functions with respect to spectral functions. These results are valid for the renormalized wave operators, which have to be introduced when the interactions are of long range.

Introduction.

In the course of deriving Hilbert-space versions of the Lippmann-Schwinger equations (1), the integral representations

\begin{align}
\Omega_{\pm}^{(\uparrow)} &= \lim_{\eta \to \pm 0} \int_{-\infty}^{+\infty} \frac{i\eta}{\mu - H + i\eta} \ d\mu F_{\mu} E_{M\pm}, \\
\Omega_{\pm}^{(\downarrow)} &= \lim_{\eta \to \pm 0} E_{M\pm} \int_{-\infty}^{+\infty} \frac{i\eta}{\lambda - H_0 + i\eta} \ d\lambda E_{\lambda}
\end{align}

of the wave operators

\begin{equation}
\Omega_{\pm}^{(\downarrow)} = \lim_{t \to \pm \infty} \exp [iHt] \exp [-iH_0 t] E_{M\pm}
\end{equation}

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have been derived. Here, \(E_\lambda\) and \(F_\mu\) are the spectral functions of the total Hamiltonian \(H\) and of the free Hamiltonian \(H_0\), respectively, and the integrals in (1) and (2) are weak Riemann-Stieltjes integrals (defined in a general context in \(^{(2)}\)) while the rest of the notation is the same as in \(^{(\ast)}\).

We have introduced in ref. \(^{(3)}\) the concept of renormalized wave operators:

\[
\Omega_\pm = \lim_{t \to \pm \infty} \exp [iHt] \exp [-iK(t; H_0)] E_{M \pm},
\]

(4)

\[
K(t; H_0) = H_0t + G(t; H_0).
\]

(5)

In the present paper we apply some of the results of \(^{(2)}\) to deriving integral representations of the type (1) and (2) for the renormalized wave operators (4). In deriving the main results of \(^{(1)}\) in this more general context we shall have the opportunity to treat in more detail some of the steps which in \(^{(1)}\) have received only casual attention, and to correct some errors which occur there \(^{(\ast)}\).

1. – Integral representations for the wave operators.

The starting point of our derivations is the set of relations

\[
\Omega_\pm = \lim_{\eta \to \pm 0} \Omega_\eta E_{M \pm},
\]

(6)

for the renormalized wave operators (4). Here we have introduced the Bochner integrals \(^{(4)}\)

\[
\Omega_\eta f = \begin{cases}
\eta \int_{-\infty}^{+\infty} \exp [-\eta t] \Omega(t) f dt \\
-\eta \int_{-\infty}^{0} \exp [\eta t] \Omega(t) dt,
\end{cases}
\]

(7)

where \(\eta \neq 0\) and

\[
\Omega(t) = \exp [iHt] \exp [-iK(t; H_0)].
\]

(8)


\(^{(\ast)}\) All the basic results of \(^{(2)}\) are correct since the mentioned errors are inconsequential in nature. Thus, the relations (2.8) and (2.24) in \(^{(2)}\) are incorrect and Lemma 2 is valid under more restricted conditions than those stated (cf. Lemma 3.1 in \(^{(4)}\), Chap. V).