V-0 Sector of the Lee Model.

II. Eigenphaseshifts and Levinson's Theorem.

J. B. BRONZAN (*) and M. CASSANDRO (**)  
Laboratory for Nuclear Science  
Massachusetts Institute of Technology - Cambridge, Mass.

M. T. VAUGHN (***)

Physics Department, Northeastern University at Boston - Boston, Mass.

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Summary. — The spectrum of the S-matrix in the V-0 sector of the Lee model is analysed. It consists, at fixed total energy, of a continuous spectrum identical with that of the disconnected S-matrix, together with a possible discrete spectrum consisting of the roots of a certain transcendental equation. This equation may have zero, one, or two roots, depending on circumstances. If at least one root exists, at all physical energies, the eigenphaseshift which joins continuously to the V-0 elastic-scattering phase shift below production threshold satisfies a Levinson theorem. The phase of the determinant of the connected S-matrix, as defined by Wright, is shown to satisfy a Levinson theorem. The situation when no stable V-particle exists is also analysed. Here the phase of the determinant of the connected S-matrix does not satisfy a Levinson theorem, but a more general result is not obtained.

1. - Introduction.

In a previous paper, one of us has given a general solution to the V-0 sector of the Lee model (1). In the present paper, we present a discussion of the spec-

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(1) M. T. VAUGHN: Nuovo Cimento, 40 A, 803 (1965), hereinafter referred to as I. The notation used here follows that of this paper, which also contains further references.
trum of the $S$-matrix, and of the possible generalizations of Levinson's theorem (2) appropriate to the model.

In Sect. 2, we examine the spectrum of the full $S$-matrix when a stable $V$-particle exists. Below the threshold for the production process

$$V + 0 \rightarrow N + 0 + 0$$

the only $S$-matrix element is the $V$-$0$ elastic scattering amplitude. Above this threshold, the $S$-matrix has a continuous spectrum which coincides with the spectrum of the disconnected $S$-matrix for the process

$$N + 0 + 0 \rightarrow N + 0 + 0$$

together with a discrete spectrum consisting of the roots of a certain transcendental equation which may have zero, one, or two roots, depending on circumstances.

Just above production threshold, the equation has two roots. One of these joins continuously onto the $V$-$0$ elastic-scattering amplitude at production threshold; the other approaches unity at the threshold. At very large energy, there is but one root of the equation, which approaches unity as the energy increases.

We cannot demonstrate the existence of at least one root at all energies; if such a root exists, however, it is the one which joins to the $V$-$0$ elastic-scattering amplitude at production threshold, and the corresponding eigenphase-shift satisfies the Levinson theorem

$$\delta(\mu) - \delta(\infty) = n\pi$$

where $n$ (= 0 or 1 here) is the number of $V$-$0$ bound states.

In Sect. 3, we show that the phase of the determinant of the connected $S$-matrix, defined by Wright (3), satisfies a Levinson theorem, with the appropriate generalization when an elementary particle coupled directly $V + 0$ is introduced into the model (4).

In Sect. 4, we examine the spectrum of the $S$-matrix when the $V$-particle is unstable. Here there is again a continuous spectrum which coincides with the spectrum of the disconnected $S$-matrix, together with a possible discrete

(2) M. T. Vaughn, R. Aaron and R. D. Amado: Phys. Rev., 124, 1258 (1961) discusses Levinson's theorem for static models; further references may be found in this paper.
