Superconvergence Sum Rules for Baryon-Antibaryon Scattering.

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Summary. — Superconvergence sum rules for baryon-antibaryon scattering amplitudes are derived assuming that the Regge pole theory describes their high-energy behaviour. We obtain superconvergence sum rules in both the forward and backward direction. Explicit expressions for the sum rules for equal-mass baryon-antibaryon scattering are given assuming that they can be saturated by meson poles of zero width.

1. — Introduction.

During the last year it has been shown (1) that some sum rules for the scattering amplitudes of the strongly interacting particles derived on the basis of current commutation relations can also be derived purely on the basis of unitarity, analyticity and high-energy behaviour of the scattering amplitudes. The asymptotic behaviour required for an amplitude $A(s, t)$ where $s = (centre-of-mass energy)^2$ is such that $\lim_{t \to 0} s A(s, t) \to 0$ for a fixed value of the momentum transfer $t$. Amplitudes which go to zero faster than $s^{-1}$ as $s \to \infty$ are called «superconvergent» and fixed momentum transfer dispersion relations for them yield the so-called «superconvergence sum rules».

To determine the asymptotic behaviour of the amplitudes we make use of the Regge pole theory of high-energy scattering. This theory in essence, tells us that properly chosen amplitudes for a scattering behave as $s^{2-n}$ in the

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purely forward or backward direction. The number \( x \) is determined by the leading Regge trajectory allowed by the quantum numbers in the appropriate cross-channel, and \( n \) represents the number of units of helicity flip.

For an amplitude \( A \) which is superconvergent, \( i.e. (x - n) < -1 \), dispersion theory yields the superconvergence sum rules (abbreviated as S.S.R.) of the form \( \int \text{Im} A(s) ds = 0 \), where it is understood that \( \text{Im} A(s) \) refers to either the backward or the forward direction. Having obtained the S.S.R. the next problem is how to exploit them. What is usually done is to saturate the S.S.R. with the contribution of a few low-lying poles and resonances. This leads to sets of equations between the masses and coupling constants of the particles involved. However, these relations suffer at present from two sources of uncertainty. Firstly, the presence of Regge cuts may spoil the conjectured superconvergent behaviour in some cases. Secondly, no clear criteria exist for choosing the «saturating set», that is, the set of states used to saturate the S.S.R. To obtain more insight into these questions it is worth-while to derive possible S.S.R. for various cases and use diverse small saturating sets to see what kind of information one can obtain.

In this note we discuss the possible S.S.R. for baryon-antibaryon scattering. The sum rules in form applicable to any baryon-antibaryon scattering are given in Sect. 2. The application of the sum rules to individual processes and the kind of information one expects to get out of them is discussed in Sect. 3. In Sect. 4, we specialize to the case of equal-mass baryon-antibaryon scattering and give the explicit relations obtained between the coupling constants of the mesons and baryons in the framework of \( SU_2 \).

2. Baryon-antibaryon scattering.

We will denote a baryon by \( B \) and its antibaryon by \( \bar{B} \). The three channels related by analyticity are

\[
\begin{align*}
(1a) & \quad \text{s-channel} \quad B_1 + B_2 \to B_1' + B_2', \\
(1b) & \quad \text{u-channel} \quad B_1 + \bar{B}_2' \to B_1' + \bar{B}_2, \\
(1c) & \quad \text{t-channel} \quad B_1 + \bar{B}_2 \to \bar{B}_2 + B_2'.
\end{align*}
\]

Goldberger et al. \( ^2 \) have shown that the invariant amplitudes \( G_i^I(s, u, t) \) where \( i = 1, 2, 3, 4, 5 \) and \( I \) is the isospin in the \( s \)-channel satisfy the Mandelstam representation. The invariant amplitudes for the \( u \)- and \( t \)-channels are denoted by \( \tilde{G}_i^u \) and \( \tilde{G}_i^t \). We follow the notation of GGMW throughout. The