The Born Series for Nonlocal Potentials (S-Wave) (*)

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Summary. — A sufficient condition for the convergence of the Born series for nonlocal potentials is derived. The analysis is restricted to the S-wave Schrödinger equation. In order to have some information on the general structure of the scattering solution as a function of the potential strength $g$ and of the linear momentum $k$, the separable potentials are reconsidered.

1. — Introduction.

The use of nonlocal potentials is required in many physical problems including the theory of nuclear matter (1) and low-energy nucleon-nucleus scattering (2).

Furthermore the solution of the Schrödinger equation for a two-body system interacting via a nonlocal potential is a nontrivial mathematical problem of nonrelativistic quantum mechanics.

The theory of nonlocal potentials is far from complete. For the most part the known results concern the so-called separable potentials, or linear combinations of separable potentials. Of course, in this case, one does not really handle

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an integrodifferential equation but simply a linear, inhomogeneous differential equation, coupled to a linear algebraic system. It is then possible to reduce the solution to quadratures.

In this paper we restrict ourselves to the S-wave Schrödinger equation, \(i.e.\) in units \(\hbar^2/2\mu = 1:\)

\[
y''(r) + k^2 y(r) = g \int_0^{\infty} V(r, s)y(s) \, ds ,
\]

where \(V(r, s)\) is supposed to be a real and symmetric function

\[
V(r, s) = V(s, r).
\]

The problem is to discuss existence and uniqueness of the scattering solution and its general behaviour as a function of the linear momentum \(k\) and of the potential strength \(g\).

We have made a first step in this direction; we have found a lower bound to the radius of convergence of the Born expansion (\(i.e.\) the representation of the scattering solution as a power series in \(g\) which can be obtained by iterating the scattering integral equation).

As is well known (\(^2\)), for a local potential satisfying the Bargmann condition, a sufficient condition for the convergence of the Born expansion is

\[
|g| \int_0^{\infty} r|V(r)| \, dr < 1 .
\]

For every value of \(g\) satisfying (1.3), the Born expansion is convergent and holomorphic in the half-plane \(\text{Im } k > 0\).

For nonlocal potentials we have found a sufficient condition very similar to (1.3):

\[
|g| \int_0^{\infty} \int_0^{\infty} s|V(r, s)| \, ds < 1 .
\]

In fact we prove that, if (1.4) is true, the scattering solution exists and is unique (in a class of functions that will be specified later); as a by-product we have that this solution can be obtained by iterating the scattering integral equation and therefore is represented as a power series in \(g\).

The new feature is that now the convergence domain in the \(k\)-variable is restricted to the real axis.