A COMPLEMENTARY PIVOTING ALGORITHM
FOR LINEAR NETWORK PROBLEMS*

M.H. SCHNEIDER

Program in Operations Research and Statistics, Department of Civil Engineering,
Princeton University, Princeton, New Jersey 08544, USA

Abstract

In this paper, an algorithm is presented for the transshipment problem that is an
adaptation of the method used by Jones, Saigal, and Schneider for solving single-
commodity, spatial-equilibrium problems. The approach uses a variable-dimension
strategy in which a sequence of subproblems is formed by solving the problem
'one-node-at-a-time'. The algorithm is tested on uncapacitated transportation
problems. Although the computational results are not directly comparable to other
methods (since the algorithm is implemented in C under UNIX), the results show
that the method is very effective and may be competitive with the best available
algorithms for linear network problems.

Keywords and phrases

Linear networks, algorithms, computational methods, primal-dual algorithm, C.

1. Introduction

Linear network problems are classical problems in operations research, logistics,
and computer science and have been the foundation for extensive theoretical and
numerical algorithmic research. The current conventional wisdom is that the fastest
general codes are based on the network simplex algorithm. Methods that are provably
polynomial, however, are based either on primal-dual methodology or the dual-simplex
algorithm. Further, the superiority of network-simplex codes is being challenged by
'primal-dual' type algorithms by Bertsekas [3] and Bertsekas and Tseng [4]. In this
paper, another non-simplex algorithm is presented, based on complementarity pivoting
methods, that also fits within the primal-dual methodology.

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The algorithm presented in this paper for the transshipment problem is an adaption of the method presented in Jones et al. [9] for solving single-commodity, spatial-equilibrium problems. In that paper, a variable-dimension approach was used to solve the linear complementarity problem which results from allowing the supplies and demands at the nodes be functions of price. A equilibrium was found by solving the sequence of subproblems formed by adding 'one-node-at-a-time'. At each step, an optimal solution for the previous subproblem was extended to an optimal solution for a subproblem with one additional node. The computational results showed that the variable-dimension approach is an effective method for solving large, structured network problems.

Here the approach is adapted for the transshipment problem. That is, given fixed supplies and demands at the nodes, find the least cost shipping plan that satisfies the demands (the underlying graph is assumed to be sparse). Although the algorithm as implemented is not polynomial, the most effective ordering of the nodes could be described loosely as a naive implementation of a scaling technique (see Edmonds and Karp [6] and Orlin [11]). With some minor modifications of the approach, a polynomial version of the algorithm could be implemented. It would be interesting to develop computational results for the best polynomial version of the algorithm.

The preliminary computational results from uncapacitated transportation problems are very encouraging and indicate that the approach may be an excellent general-purpose method for linear network problems. It must be noted, however, that the computational work has been programmed in C under the Berkeley 4.2 UNIX operating system and, therefore, is not directly comparable with previous computational research. Future computation experiments will be performed to determine if the algorithm performs as well on general minimum-cost flow problems.

The problem is formulated and the necessary definitions are given in sect. 2. The mechanics of pivoting are developed in sect. 3 and incorporated into the algorithm in sect. 4. In sect. 5, the algorithm is extended to handle infeasibility. Some of the details of the implementation, including heuristics for ordering the nodes, and the numerical results are presented in sect. 6. Section 7 concludes with some possible extensions of this work to be considered in the future.

2. Problem formulation and background

The approach taken is to assume that an optimal solution for nodes \{1, 2, \ldots, K - 1\} is given. An optimal solution for nodes \{1, 2, \ldots, K\} is computed using the \(K - 1\) node optimal solution as the initial solution for the \(K\) node subproblem. The incoming node has either an initial deficit (if it is a demand node) or an initial surplus (if it is a supply node). A sequence of pivots is performed, strictly reducing the deficit (or surplus), until a new optimal solution is reached.

Any ordering of the nodes can be used, although computationally the ordering is critical to the overall performance of the algorithm. The ability to use an arbitrary