Quasi-Static Waves in Inhomogeneous Magnetoplasma Slabs (*).

P. de Santis (**)  
Selenia S.p.A. - Roma  
(ricevuto il 18 Luglio 1964)

Summary. — Starting from Maxwell's equations, a theoretical investigation of the propagation of a quasi-static E-mode in a longitudinally magnetized plasma slab is made. A transmission-line equation is obtained for the electric field and the Ritz-Rayleigh variational method is used to minimize the error over the cross-section of the slab, when a trial function for the field distribution in the plasma is assumed. A parabolic distribution is taken as an analytical model for the electron-density profile in the cross-section, and the electron density is supposed to vanish at a point outside the plasma slab; in the limit for the zero point to go to infinity, uniform case-equations are recovered. Under these conditions, a dispersion relation is derived and dispersion curves are calculated and plotted for some numerical cases of interest. In the homogeneous case a comparison is made between the approximate solutions and the exact solutions.

List and explanation of main symbols.

$x, y, z$ co-ordinate axes, as in Fig. 1,
$E$ electric field of the propagating wave,
$H$ magnetic field of the propagating wave,
$E_{zp}(u)$ cross-sectional distribution of the longitudinal electric field component inside the plasma,

(*) This work has been sponsored by the Cambridge Research Laboratories, OAR through the European Office, Aerospace Research, United States Air Force under Contract AF 61(052)-145.

$E_{z\phi}(u)$ cross-sectional distribution of the longitudinal electric field component outside the plasma,
$E_0, E_1$ amplitude constants of $E_{z\phi}(u)$,
$\beta$ longitudinal propagation constant,
$u$ $\beta x$,
$2d$ slab thickness,
$2D$ distance between outside parallel metal plates,
$l$ value of $|x|$ (> $d$) at which the electron density is zero,
$B_0$ longitudinal static magnetic field,
$\varepsilon$ dielectric constant tensor,
$\varepsilon_1, \varepsilon_2, \varepsilon_3$ dielectric constant tensor components,
$\mu_0$ free space permeability,
$\delta$ $\beta d$,
$\Delta$ $\beta D$,
$\lambda$ $\beta l$,
$f(u)$ cross-sectional, symmetric distribution of the electron density, normalized so that $f(0) = 1$,
$L$ second-order linear differential operator,
$\mu$ $\lambda/\delta$,
$\mu$ parameter characterizing the cross-sectional inhomogeneity (>1),
$\omega$ angular frequency of the propagating wave,
$\omega_b$ electron cyclotron angular frequency at the $B_0$ field,
$\omega_{\phi0}$ plasma angular frequency at the $x=0$ center of the slab,

$$X_1 = \omega_b^2/\omega^2 = 1/\delta^2,$$
$$Y^2 = \omega_b^2/\omega^2,$$
$$a = \omega_b^2/\omega_{\phi0}^2,$$
$$\tau = 1 - (1 - Y^2)/X_0,$$
$$\tau_0 = 1 - 1/X_0.$$

1. - Introduction.

The propagation of electromagnetic waves in a magnetoplasma for which the average net charge is zero, may be macroscopically characterized by an equivalent dielectric constant, $\varepsilon$, which, in the most general case, is a tensor quantity.

In a lossless case, if the plasma is cold so that pressure and temperature gradients are neglected, the dielectric tensor is hermitian and its structure is

$$\varepsilon = \begin{pmatrix}
\varepsilon_1 & j\varepsilon_2 & 0 \\
-j\varepsilon_2 & \varepsilon_1 & 0 \\
0 & 0 & \varepsilon_3
\end{pmatrix}.$$