Non-Linear Effects in the Vacuum Polarization.

A. MINGUZZI

Istituto di Fisica dell'Università - Bologna
Istituto Nazionale di Fisica Nucleare - Gruppo di Bologna

(riccetto il 4 Giugno 1956)

Summary. — We have investigated the vacuum polarization current arising from the superposition of a constant electromagnetic field \( \mathbf{e} \cdot \mathbf{h} = 0 \), \( \mathbf{h}^2 - \mathbf{e}^2 > 0 \), with a small arbitrarily varying field, without resorting to a power series development in the constant field. The energy density of a photon field in the constant field is also given.

1. Introduction.

In this paper we have investigated the vacuum polarization current arising from the superposition of a small arbitrarily varying electromagnetic field with a constant electromagnetic field, which in a particular frame becomes a constant magnetic field, \( \mathbf{h}_{12} \), pointing in the \( x_3 \)-direction. The effect of the latter field consists only in a modification of the vacuum current caused by the former field. This is because a magnetic field is incapable of polarizing the matter vacuum. In this specific situation one can calculate the vacuum current without resorting to its series development in the magnetic field. Some difficulties met in discussing the gauge invariance properties of the vacuum current, have been completely removed using an extension of a procedure suggested by J. Schwinger (1). The vacuum current can be expressed covariantly as sum of three terms, one of which depends only on the current associated with the varying field and the other two depend in a gauge invariant manner on the derivatives of the same field. The charge renormalization procedure affects only the first term. If the small field is interpreted as the quantized field associated with a photon field, the current terms depending on the derivatives give rise to additional terms in the Hamiltonian which describe the coupling between the photons and the magnetic field.

The full Hamiltonian will be invariant with respect to translation in space and time, but not invariant with respect to rotations around the axis forming a non vanishing angle with the magnetic field direction. If, for instance, a photon with a definite linear momentum and definite circular polarisation enters the magnetic field region perpendicularly, it will leave out the magnetic field region as a mixture of the two opposite circular polarizations. But this is possible only if the two components of the initial circular polarization, polarized parallelly and perpendicularly to the magnetic field, propagate with different velocities in the magnetic field region.

Therefore, the magnetic field region will behave for a light wave as an anisotropic dielectric.

2. - Calculation of the Polarization Tensor $K_{\mu\nu}(x, x')$.

We have first attempted the calculation using the «bound interaction picture» in which the «bounded field» is the constant magnetic field. No difficulty arises in connection with the vacuum definition, since the negative and positive energy levels of the corresponding Dirac equation have a spacing of $2m^2$. An entirely different situation would have arisen if we had started with a pure electric field (2). In such a case the negative energy levels of the Dirac equation go continuously into the positive ones (2).

We have chosen this particular gauge associated with the magnetic field $h_{11}$

$$a_1(x) = -\frac{1}{2}h_{12}x_2; \quad a_2(x) = \frac{1}{2}h_{12}x_1; \quad a_3 = a_\phi = 0.$$ 

The vacuum current is correspondingly:

$$i\lim\limits_{x' \to x} \{ T\{ \gamma_a \lbrack \partial_s - iea_s(x') \rbrack - m \} A^{(1)}(x', x) \} = 0.$$ 

By adding a small arbitrarily varying field $A_\mu(x)$ the vacuum current becomes

$$\delta j_\mu(x) = \frac{i}{2} \int K_{\mu\nu}(x, x') A_\nu(x') d^4x',$$

with

$$K_{\mu\nu}(x, x') = \langle [\gamma_\nu(x'), j_\nu(x')]_0 \rangle e(x, x') =$$

$$= -\frac{i\epsilon}{2} \{ \gamma_\nu \gamma_s \lbrack \partial_s - iea_s(x') \rbrack - m \} \Delta(x, x') \gamma_s \{ \gamma_\nu \lbrack \partial_s - iea_s(x') \rbrack - m \} \Delta^{(1)}(x', x) + \mu \to \nu \},$$