Against the Necessity of a Three-Dimensional Time.

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In the recent times one can notice in the literature a flourishing of papers in which the time is considered as a three-dimensional vector (1-3).

A first paper against this physical theory is the one by STRNAD (4). He observes that in such a theory the transversal Doppler effect is missed completely whilst such an effect has been measured.

Here I will consider a theoretical topic only. In some of those papers (2,3) the necessity of such a theory is asserted due to a presumed inconsistency of the standard kinematics. It seems to me that such an inconsistency does not exist at all and that therefore, at least from this point of view, no need for a new kinematics is present.

In two papers (2,3) the inconsistency of the following postulates of special relativity is asserted:

a) ordinary space is three-dimensional and Euclidean,

b) light speed is invariant.

The objection of ref. (2) is that a) implies:

i) Any (pointlike) object which is free (i.e. subjected to no force) has three degrees of freedom when moving in (empty) space. Namely, in order that its (instantaneous) spatial position may be univocally determined, it always needs to be assigned a number of three independent functions of time.

ii) The motion of any free (pointlike) object can always be characterized by three independent (Cartesian) velocity components whose mutual orthogonality requires any variation of each one of them to cause by itself no change in the remaining two components.

A first minor remark is that the definition of freedom based on the absence of forces acting on the pointlike object is at least strange in view of characterizing its degrees of

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freedom. The same remark applies to the requirement of motion in empty space. The concept of degrees of freedom pertains to kinematics, i.e. to the geometry of the movement. It is a theory in which the primitive quantities are length and time. Therefore the concept of force and of density of matter are completely extraneous to such a theory.

Apart from these minor remarks, in ref. (3) it is asserted that neither i) nor ii) would seem to be fully satisfied by the standard kinematics of any free photon. In short, after noticing that the photon travels with light speed, it is said that such a constraint should lower the number of degrees of freedom. More explicitly, the example of a photon in a plane is taken. After setting the components of the velocity as $c_x = c \cos \varphi$ and $c_y = c \sin \varphi$ the statements (2.3) are that the co-ordinates $x(t; c_x)$, $y(t; c_y)$ are both determined as soon as the mere co-ordinate $\varphi$ is known and the two required photon degrees of freedom on the $xy$-plane would actually reduce to one alone, represented by $\varphi$.

At this point it seems worthwhile to recall the existence of nonholonomic constraints. It is only the holonomic constraint that reduces the number of the possible configurations of the system (here the point. On the contrary, a nonholonomic constraint does not reduce the number of the possible configurations; in other words the system may reach the same configurations as it could before applying the constraint. What is changed is the way through which the system can reach a given configuration. That is why the nonholonomic constraint are also called constraints of pure mobility.

The constraint

$e_\varphi^2 + c_\varphi^2 = c^2 = \text{invariant}$,

which is eq. (2) of ref. (2.3), is in fact a nonholonomic constraint since it is a differential equation in the co-ordinates and cannot be put in finite terms by an integration.

For example one could say that the photon in the plane would have one degree of freedom only, if the position of the photon at a certain time $t$ would be given once the value of the variable $\varphi$ is known at the same time $t$. But it is not so. Because of the nonholonomic constraint the position of the photon at the time $t$ is known only if the value of the function $\varphi(t)$ is known during the interval $t_0 \rightarrow t$ and the position of the photon is given at $t_0$.

The same happens with a pointlike particle in special relativity. In this ease the constraint is given by

$\dot{x}^\alpha \dot{x}^\beta \eta_{\alpha\beta} = c^2$,

where $(x^\alpha)$ is a generic co-ordinate system, $\dot{x}^\alpha = dx^\alpha/ds$, $ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta$, and $\eta_{\alpha\beta}$ is the fundamental metric tensor of the pseudo-Euclidean space-time reducing, in orthogonal Cartesian co-ordinates to $\eta_{\alpha\beta} = \text{diag}(+1, -1, -1, -1)$. In other words, eq. (2) says that the modulus of the four-velocity of the particle is always equal to the light speed $c$.

Again, the constraint (2) is nonholonomic and therefore it is not true that in the ordinary relativistic kinematics of a pointlike particle it would reduce the degrees of freedom $\varphi$ (such a conclusion is given in ref. (2.3) on the basis of equations which are a direct consequence of eq. (2)).

Coming back to the statements (3) i) and ii), it seems that the confusion lies in the fact that in special relativity the pointlike object or the photon are not free but they are subjected to a nonholonomic constraint. This does not reduce the number of the possible configurations but invalidates the statement ii). As to the latter, it (or a modified version) cannot be a direct consequence of the statement a). Indeed the be-