Dynamical Generation of the Time-Dependent Poisson Distribution for Particle Production.

II: Stimulated Decay (*).

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Summary. -- In an earlier installment a dynamical derivation of the time-dependent Poisson distribution for particle production, say, of a decaying body was derived by a Lagrangian formulation as a more fundamental approach to the standard one which relates the probability of emission of \( n \) particles to that of emission of \( n - 1 \) particles. This dynamical derivation is generalized further to study stimulated decaying processes by a decaying body during its active lifetime \( t \), where some particles may be present initially and the latter get absorbed by the body before it starts to decay and further decay may be stimulated.

In an earlier note (1) a dynamical generation for the time-dependent Poisson distribution, for a decaying body, which is particularly well known in nuclear physics, was derived based on a Lagrangian formulation as a more fundamental approach to the standard one relating the probability of emission of \( n \) particles to the probability of emission of \( n - 1 \) particles. In the Lagrangian formulation (second quantized approach), we may write

\[
L(x) = -\frac{i}{2} (\phi^* \phi - \phi^* \phi) - \frac{1}{2m} \nabla \phi^* \cdot \nabla \phi + K^* \phi + K \phi^*,
\]

where \( K(x) \) is an external (c-number) source playing the role of the decaying body, \( x = (x_0, x) \). The vacuum-to-vacuum transition amplitude is given by the well-known formula:

\[
(1) \quad \langle 0_+|0_-\rangle^K = \exp \left[ i \int (dx)(dx') K^*(x)G(x \to x')K(x') \right] = \exp \left[ iK^* GK \right],
\]

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where
\begin{align}
(2) \quad G(x - x') &= i \int \frac{d^3k}{(2\pi)^3} \exp[ik \cdot (x - x')] \exp[-ik^2(x^0 - x'^0)/2m], \quad \text{for } x^0 > x'^0, \\
(3) \quad G(x - x') &= 0, \quad \text{for } x^0 < x'^0.
\end{align}

We consider the following general class \(^{(1)}\) of sources:
\begin{align}
(4) \quad K(x) &= i^j h_t(x^0) K(x),
\end{align}
where \(h_t(x^0) = 1\) for \(-t/2 < x^0 < +t/2\), and is zero otherwise, and
\begin{align}
(5) \quad K(x) &= \int \frac{d^3k}{(2\pi)^3} F(t^1k) \exp[ik \cdot x],
\end{align}
and we impose the following integrability conditions:
\begin{align}
(6) \quad \int \frac{d^3Q}{(2\pi)^3} |H(Q)|^i < \infty, \quad i = 1, 2,
\end{align}
where
\begin{align}
(7) \quad H(Q) &= F(Q) \frac{\sin(Q^2/4m)}{Q^2/2m}.
\end{align}

Then one obtains \(^{(2)}\) from (1), in particular, by using \(^{(1-3)}\) unitarity and completeness relations, with respect to the external sources, in the process that the probability of emission of \(n\) particles during the active lifetime \(t\) of the body:
\begin{align}
P_n(t) &= \frac{(\lambda t)^n}{n!} \exp[-\lambda t],
\end{align}
where \(\lambda\) is independent of \(t\), denotes the average number of particles emitted per unit time and is given by
\begin{align}
(8) \quad \lambda &= 4 \int \frac{d^3Q}{(2\pi)^3} |F(Q)|^2 \frac{\sin^2(Q^2/4m)}{(Q^2/2m)^2} \quad (< \infty)
\end{align}
with \(\lambda t = \sum_{n=0}^\infty nP_n(t)\).

To study stimulated decay processes, we write \(^{(2)}\): \(K(x) = K_1(x) + K_2(x) + K_3(x)\), where \(K_2\) is switched on after \(K_1\) is switched off, and \(K_3\) is switched on after \(K_2\) is switched off. The vacuum-to-vacuum transition amplitude (1) may be then rewritten as
\begin{align}
(9) \quad \langle 0_+ | 0_- \rangle^K &= \langle 0_+ | 0_- \rangle^{K_3} \langle 0_+ | 0_- \rangle^{K_1} \exp[iK_3^* iK_2] \exp[iK_2^* iK_1] \exp[iK_1^* iK_1],
\end{align}