Klein Paradox of the Second Kind.

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Summary. - In this note we discuss how a nonrelativistically nonconfining potential gives confinement when it is used as a scalar in the Klein-Gordon or Dirac equation.

It is well known that the Coulomb potential gives pure bound states when used in the Schrödinger equation. It is also known that it gives pure bound states when used in the Klein-Gordon (KG) or Dirac equation (1).

It is well known that a linearly rising potential gives pure bound states (i.e. confines the particle on which it acts) when used in the Schrödinger equation. However, when this potential is used in the KG or Dirac equation as the fourth component of a four-vector, it gives nonconfinement (2). This phenomenon is generally known as Klein paradox (3). On the other hand, if the linearly rising potential is used as a Lorentz scalar in the KG or Dirac equation, it gives confinement, i.e. pure bound states (2,4).

It is for this reason that relativistic quark models have been built on the concept of a scalar potential (5).

But let us consider the spherically symmetric potential

\[ V_s(r) = -\alpha r, \]

\[ \alpha > 0. \]

(1)

(2) See, for example, B. Ram: Am. J. Phys., 50, 549 (1982).
(4) In fact, any polynomial potential \( V_s(r) = \sum_{n=1}^{\infty} a_n r^n \) gives confinement when used as a scalar in the KG or Dirac equation; see B. Ram: Lett. Nuovo Cimento, 28, 476 (1980); and M. Plesset: Phys. Rev., 41, 278 (1932).
Potential (1) decreases linearly and approaches $-\infty$ as $r$ goes to infinity. It is obviously nonconfining (i.e., does not give pure bound states) when used in the nonrelativistic Schrödinger equation. However, when used in the KG equation as a scalar it gives confinement. This is seen below.

The KG equation with a spherically symmetric scalar potential $V_s(r)$ is (6)

$$[-\nabla^2 + (m + V_s(r))]\psi(r) = E^2\psi(r).$$

Equation (2) can be rewritten in the Schrödinger form

$$[-\nabla^2 + V_{\text{eff}}(r)]\psi(r) = E\psi(r)$$

with

$$\bar{E} = E^2 - m^2$$

and

$$V_{\text{eff}}(r) = 2mV_s(r) + V_s^2(r).$$

With $V_s(r)$ given by (1),

$$V_{\text{eff}}(r) = -2mr + z^2r^2.$$

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Fig. 1. $-\frac{1}{2}V_{\text{eff}}(r)$ of eq. (6) (with $m = \frac{1}{2}$, $z = 1$) vs. $r$. $V_{\text{eff}}(r) \to +\infty$ as $r \to \infty$.

(*) We use the natural units $\hbar = c = 1$. 