Single-Particle Model and EMC Effect.

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Summary. – The famous EMC effect is usually discussed in terms of quarks and gluons. One of the possible interpretations implies that the radius of a nucleon within a nucleus is larger than the one of a free nucleon. Already the naive single-particle model is shown to predict such a "medium" effect; in some cases with distinct differences in the neutron and proton radii.

Motivation. The recent (1) European Muon Collaboration’s (EMC) result that the deep inelastic structure functions of iron and deuterium are different appeared to nuclear and particle physicists as a surprise. Subsequent attempts to explain this so-called EMC effect led to a vast number of papers. Doing injustice to quite a lot of them we refer only to ref. (2-6) which could serve as sources for further work.

Most of the contributions presented in the very lively discussion of this topic rely on a "microscopic" picture invoking quarks and gluons. But to-date none of the proposed interpretations has been accepted as the best and most consistent one. Hence, we infer that there is still room for studies adopting a different point of view at the subject. Below a "macroscopic" discussion of the EMC effect is presented, i.e. one involving only nuclei and nucleons.

To that end we employ the naive (yet, extremely successful) single-particle model. It will be shown to predict nucleon radii within nuclei that are larger than the ones

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of free nucleons. According to (5) this is a possible explanation for the EMC effect
(a notion which did not remain undisputed (6) like almost all available interpretations).

**Preliminaries.** From (7) it is recalled that the volume integral over the shell-model
potential, \( U(r) \), divided by the number of nucleons, \( A \), contained in the nucleus under
consideration, is given by

\[
J_v(A) = - \left( \frac{8\pi\hbar}{\sqrt{2m}} \right) \cdot \sum_i n_i \sqrt{-E_i/A}.
\]

\( E_i < 0 \) is the single-particle energy eigenvalue and \( n_i = 2j + 1 \) is the degeneracy of
the \( i \)-th level. The summation is to be performed separately over all occupied (ground-
state) neutron and proton levels. In terms of the respective volume integrals per pro-
ton, \( J_p^p(A) \), and per neutron, \( J_n^n(A) \), we have

\[
J_v(A) = \frac{1}{2} \left[ J_p^p(A) + J_n^n(A) \right] = - \left( \frac{8\pi\hbar}{\sqrt{2m}} \right) \cdot \frac{1}{2} \sum_k \sqrt{-E_{kr}/Z} + \frac{1}{2} \sum_{l=1}^{N_p} n_l \sqrt{-E_{vl}(A-Z)}.
\]

\( Z \) stands for the number of protons in the respective nucleus and \( N_p(N_n) \) is the number
of occupied proton (neutron) levels. For further details on (1) and (2) ref. (7) should
be consulted.

In the upper part of fig. 1 the respective volume integrals are plotted vs. the mass
number \( A \). In contrast to the smooth behaviour of \( J_v(A) \), the \( J_p^p(A) \) and \( J_n^n(A) \) exhibit
for \( A > 60 \) rather large fluctuations. The global features of \( J_v(A) \) remind us of the
saturation of nuclear forces as, e.g., reflected by the mass number dependence of the
binding energy per nucleon, \( B(A) < 0 \), of the atomic nuclei. Indeed, this impression
is correct. The discussion of numerical results for \( J_v(A) \) based on (1) lead to the
formula

\[
J_v(A) \approx - \left( \frac{8\pi\hbar}{\sqrt{2m}} \right) \cdot \sqrt{-B(A)/N(A)} , \quad N(A) \approx 0.38 ,
\]
as a rather good approximation to (1), see also (7). As demonstrated in the middle of
fig. 1, the heuristic coefficient \( N(A) \) is almost constant with respect to variations in
the mass number \( A \).

**Discussion of the nucleon radii.** Let us make the simplifying, yet, fairly reasonable
assumption that each nucleon within a nucleus occupied a spherical volume with the
radius \( R(A) \). This leads in view of the foregoing directly to the relation

\[
J_v(A) = U_0(A) \cdot R^3(A) \cdot 4\pi/3.
\]

\( U_0(A) \) is the depth of the respective shell model potential, i.e. the only quantity in (4)
which is characteristic for the forces acting in the system under consideration. Solving (4)
for \( R(A) \) and using \( R(D) = R(2) \) for the radius of a nucleon within the deuteron, we
define now the relative radius \( r(A) \) via

\[
r(A) = \frac{R(A)/R(D)}{\left( [J_v(A)/J_v(D)] [U_0(D)/U_0(A)] \right)^1/2}.
\]