Consistency of QCD Sum Rules and Predictions for $g_{\pi NN}$, $f_\pi$ and $m_\pi$.

J. Dey

Laboratoire de Physique nucléaire, Université de Montréal
C. P. 6128, Succ. A 8, Montréal, Qué., H3C 3J7, Canada

P. Ghose

British Council Division, British High Commission - 7, Shakespeare Sarani, Calcutta, India

(ricevuto il 29 Marzo 1985)

PACS. 11.15. – Gauge field theories.

Summary. – A new constraint on $f_\pi$ is obtained from consistency of QCD sum rules for two- and three-point functions. This yields the remarkable result $g_{\pi NN} = 4\pi$ in the absence of power corrections and leads to surprisingly good values for $f_\pi$ and $m_\pi$.

The application of the techniques developed by Shifman, Vainshtein and Zakharov (1) has led to numerous sum rules in QCD which have greatly improved our understanding of hadronic physics. Following these techniques for three-point functions, Reinders, Rubinstein and Yazaki (2) have calculated the pion-nucleon coupling constant, $g_{\pi NN}$, which happens to involve the pion decay constant $f_\pi$ in a role reverse to that in the Goldberger-Treiman relation. The latter relation can also be obtained using similar techniques with two-point functions (3). Consistency therefore imposes a new constraint on $f_\pi$. This leads to a remarkable parameter-free result $g_{\pi NN} = 4\pi$ when nonperturbative power corrections are neglected. It also determines the pion mass, $m_\pi$, in terms of only $m_a$, the current quark mass in the QCD Lagrangian and the quark condensate $\langle 0|\bar{q}q|0\rangle$.

The starting point is the current which has all the quantum number of the nucleon

(1) $\eta_{\Lambda}(X) = \varepsilon_{abc}(w^a(X)C\gamma_{\mu}w^b(X))\gamma_5\gamma^\mu d^c(X),$

where $u$ and $d$ are the quark fields, $a, b, c$ are the colour indices and $C$ is the charge conjugation operator. A second choice for the current is to replace $\gamma_{\mu}$ with $\sigma_{\mu\nu}$. However,

(3) L. J. Reinders: QCD sum rules: an introduction and some applications, CERN preprint TH3701, Lectures presented at 23rd Cracow School of Theoretical Physics, Zakopane (1983).
it has been argued by Ioffe (4) that chiral-symmetry breaking terms are strongly suppressed in the polarization of this current and consequently the resonance should not couple strongly to this current. Taking $\lambda_N$ to be the phenomenological coupling of the nucleon to this current, one gets the two results for the self-energy (3,4)

\[ 2aM^4 = 2(2\pi)^4 \lambda_N^2 \exp \left( - \frac{M_N^2}{M^2} \right), \]

where

\[ a = - (2\pi)^2 \langle 0 | \bar{q} q | 0 \rangle \]

and

\[ M^6 X = 2(2\pi)^4 \lambda_N^2 \exp \left( - \frac{M_N^2}{M^2} \right), \]

where

\[ X = 1 + \frac{4}{3} a^2 / M^6 + \text{gluon condensate contributions}. \]

Using three-point functions it was shown by Reinders et al. (2) that

\[ \lambda_N^2 \exp \left( - \frac{M_N^2}{M^2} \right) \frac{M_N}{M^4} \frac{f_\pi}{\sqrt{2}} \frac{M_N^2}{M^2} = \frac{1}{(2\pi)^3} M^2 \langle - 4m_0 | \bar{q} q | 0 \rangle \cdot \]

Using (6) and (4) they obtained the remarkable result

\[ g_{\pi N N} = 2(2\pi)^2 \sqrt{2} f_\pi \frac{1}{M_N \sqrt{X}}. \]

One interesting feature of this relation is that it is almost independent of $M^2$, the Borel transform mass scale. The other feature is the inverse dependence on $f_\pi / M_N$ compared to the Goldberger-Treiman relation

\[ g_{\pi N N} = \sqrt{2} M_N / f_\pi, \]

which also follows (2) from consideration of two-point functions for $g_{\pi N N}$ with a phenomenological pion-nucleon Lagrangian. If one assumes nucleon pole dominance, $M \approx M_N$ and $X \approx 1.4$ for $a = 0.62$ (GeV)$^3$.

It follows from (7) and (8) that

\[ f_\pi / M_N = \sqrt{X} / (2\pi \sqrt{2}) . \]

This is a constraint which has not been specifically noticed and used in the literature. Using (9) in (7) we get

\[ g_{\pi N N} = 4\pi / \sqrt{X}. \]

Comparing the standard current algebra relation

\[ f_\pi^2 = - 4m_0 \langle 0 | \bar{q} q | 0 \rangle / m_N^2, \]

---