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Field Operators and Retarded Functions.

V. Glaser

Institute « Rudjer Boskovic » - Zagreb

H. Lehmann

Institut für theoretische Physik, Universität Hamburg

W. Zimmermann

Max Planck-Institut für Physik - Göttingen

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Summary. — A formula for the expansion of field operators with respect to products of ingoing fields is derived. The coefficients in this expansion are the retarded functions introduced earlier. They fulfil a system of equations which turns out to be not only a necessary but also sufficient condition for such functions to yield a local field operator.

We discuss some consequences of a recent formulation of quantized field theories by means of retarded products (1). First we derive a formula for the expansion of field operators with respect to products of incoming fields which is not restricted to perturbation theory. The following section contains a system of equations for the retarded functions. It serves to state necessary and sufficient conditions for these functions, so that the general principles of field theory are fulfilled. Our results are valid only for theories without stable bound states. For the latter case we refer to a paper by Nishijima (2).

(1) H. Lehmann, K. Symanzik and W. Zimmermann: *Nuovo Cimento* (in press). In the following quoted as LSZ II.

(2) K. Nishijima: to be published.

1. – Expansions with respect to incoming fields.

HAAG [3] has shown under very general assumptions that a field operator $A(x)$ may be expanded in the following manner:

$$A(k) = \delta(k^2 + m^2)A_{\text{in}}(k) + \sum_{n=2}^{\infty} \frac{1}{n!}\int \mathrm{d}k_1 \dots \mathrm{d}k \; \frac{\delta(k - \sum k_i)}{k^2 + m^2 - i\varepsilon k_0} \cdot \tag{1}$$

$$\cdot g(k_1, \dots, k_n)\, \delta(k_1^2 + m^2) \dots \delta(k_n^2 + m^2) \colon A_{\text{in}}(k_1) \dots A_{\text{in}}(k_n)$$

$A(k)$ denotes the Fourier transform of $A(x)$:

$$A(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \mathrm{d}k \exp\,[+\,ikx] A(k)\,. \tag{2}$$

The function $g(k_1, \dots, k_n)$ are defined only for $k_i^2 + m^2 = 0$. We shall derive such an expansion using only the asymptotic condition at $t = -\infty$. Haag's functions $g(k_1, \dots, k_n)$ turn out to be closely related to the retarded functions [1]

$$\begin{cases} r(x;\, x_1 \dots x_n) = (\Omega,\, R(x;\, x_1 \dots x_n)\Omega)\,, \\ R(x;\, x_1 \dots x_n) = \\ \qquad = (-\,i)^n \sum \theta(x - x_1) - \theta(x_{n-1} - x_n)[\dots[A(x),\, A(x')] \dots,\, A(x_r)]\,. \end{cases} \tag{3}$$

We note first that any linear operator L, operating in the Hilbert space of the basis vectors $\{\Phi_{\text{in}}^{q_1 \dots q_k}\}$, allows the expansion [4]

$$L = \sum_{n=0}^{\infty} \frac{1}{n!}\int \mathrm{d}k_1 \dots \mathrm{d}k \; (\Omega,\, [\dots[L,\, A_{\text{in}}^*(k_1)] \dots A_{\text{in}}^*(k_r)]\Omega) \cdot \tag{4}$$

$$\cdot \varepsilon(k_1)\, \delta(k_1^2 + m^2) \dots \varepsilon(k_n)\, \delta(k_n^2 + m^2) \colon A_{\text{in}}(k_1) \dots A_{\text{in}}(k_n) \colon,$$

since both sides of this relation coincide for arbitrary matrix elements of the system $\{\Phi_{\text{in}}^{q_1 \dots q_k}\}$. In the particular case that L is the field operator $A(x)$ terms with $n = 0, 1$ are given by

$$(\Omega,\, A(x)\,\Omega) = 0\,,$$

$$\int \mathrm{d}k(\Omega,\, [A(x),\, A_{\text{in}}^*(k)]\,\Omega)\, \varepsilon(k)\, \delta(k^2 + m^2)\, A_{\text{in}}(k) = A_{\text{in}}(x)\,.$$

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[3] R. HAAG: *Dan. Mat. Fys. Medd.*, **29**, 13 (1955).

[4] The infinite sums in (4) and (5) do not lead to problems of convergence, since all matrix elements with respect to incoming states have only a finite number of non-vanishing terms.

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