We are interested by the inelastic scattering of electrons by nucleons or nuclei

\[ e^- + A \rightarrow e^- + B + C, \]

and we try to relate this process to the associated photonuclear reaction

\[ \gamma + A \rightarrow B + C, \]

by using first order calculations in the electromagnetic coupling constant

\[ \alpha = \frac{e^2}{4\pi} = \frac{1}{137}. \]

1. Let us consider the general process \( e^- + A \rightarrow e^- + B + C \).

The \( S \) matrix element is given by

\[ S_{ij} = \delta_{ij} \cdots \frac{i}{(2\pi)^4} \delta_4(k + p_A - k' + p_B - p_C) \left( \frac{m_e^2}{k \cdot k'} \right) \left( \frac{N_A N_B N_C}{p_A^0 p_B^0 p_C^0} \right) \Gamma_{ij}, \]

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where $m_e$ is the electron mass and $N_i$ the normalization coefficients defined by

$$N_i = M_i$$

for fermions of mass $M_i$ and $N_j = 1$ for bosons.

The differential cross-section is given in terms of the matrix element $T_{fi}$ by

$$d\sigma = \frac{1}{(2\pi)^5} \frac{m_A^2 N_A N_B N_C}{m^2 M_\Lambda} \left| T_{fi} \right|^2 \frac{dP_A dP_B dP_C}{k_0} \frac{dP_A^* dP_B^* dP_C^*}{k'} \frac{1}{(k - p^* - p_B - p_C - k')^2}$$

2. We use the first Born approximation for the electromagnetic interaction and we replace the general diagram of Fig. 1 by the following one:

![Figure 2](image)

The $T$ matrix is the product of two matrix elements for the electromagnetic current $j_{\mu}$:

$$T_{fi} = \frac{1}{q^2} \langle k' | j_{\mu} | k \rangle \langle BC | j_{\mu} | A \rangle$$

The first one is simply

$$\langle k' | j_{\mu} | k \rangle = i \bar{u}_e(k') \gamma_\mu u_e(k),$$

where $u_e(k)$ is a free Dirac spinor describing an electron of energy momentum $k$ and spin $s$.

The second one describes the photoreaction $\gamma + A \rightarrow B + C$ for a virtual photon and is unknown.

3. We have now to calculate $|T_{fi}|^2$ with the factorized form (2):

$$|T_{fi}|^2 = \frac{1}{q^4} \sum_s \frac{1}{2} \left| \langle k' | j_{\mu} | k \rangle \langle k' | j_{\nu} | k' \rangle^* \right|^2 \left| \frac{1}{2s_A} + \sum_{s_B s_C} \langle BC | j_{\mu} | C \rangle \langle BC | j_{\nu} | C \rangle^* \right|.$$