On Particle Creation by a Supercritical External Field.

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Summary. - We propose an approach to particle creation in a supercritical external field as a scattering process with semi-classical back reaction when the field is switched on. We show how this approach would work on a simple, though not realistic model and compare it to the usual one.

It is known, in quantum field theory, that particles can be created out of vacuum under the action of an external field (1).

In general, this happens with a nonstatic external field (we do not consider fields of infinite extent or with a horizon).

But recently, interest arose for a creation process where there is a static but so strong external field that the gap, between positive- and negative-energy modes, closes; and some of the modes become resonances, with a complex energy (2). The study of this phenomenon has been done along an idea given by Gersten and Zel'dovich (6): the presence of complex modes suggests to consider the particle creation as an autoionization (or decay) of the vacuum. But one can give an example of a strong-field situation where there are no complex modes; for instance take the Dirac field describing neutrons with a given magnetic moment μ, in a uniform magnetic field $H$, one finds easily that the modes satisfy

$$\left[k_0^2 - k^2 - ((m^2 + k^2 + k^2)^2 + \mu H)^2 \right] \left[k_0^2 - k^2 - ((m^2 + k^1 + k^2)^2 - \mu H)^2 \right] = 0$$

($H$ is along the 3 axis, 0 denotes the time axis) for $H > m/\mu$ the gap closes, but $k_0$ remains real; of course actual, neutrons, being composite will behave in a more complicated way.

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for the high \( H \) values involved, nevertheless this suggests to ask whether there is particle creation in such a case.

The simplest example with that behaviour is a neutral, scalar, particle described by the field \( \mathcal{V} \) in an external uniform field \( V \) whose equation is

\[
\partial^2 \partial_t \psi + (m - V)^2 \psi = 0,
\]

though it is not realistic, we shall use it to describe the process of particle creation using only the closure of the gap.

To do that, instead of trying to quantize \( \mathcal{V} \) beyond the critical value \( V = m \), we shall follow what happens when \( V \) is switched on. Then, there are two important points.

1) In any experiment, the external field is time dependant during the switching on, if it is slow enough the initial vacuum is maintained as long as \( V < m \); but if we try to reach the supercritical value \( V > m \), one necessarily violates, the adiabaticity condition \( \partial \kappa_0 / \partial t \ll \kappa_0^2 \), however slow \( V(t) \) is. The same applies if complex \( \kappa_0 \) values appear, for instance in a model with \( \kappa(t) = \kappa_0(t) \) instead of \( (m - V)^2 \).

2) The actual \( V(t) \) comes from a part \( V^{\text{ext}} \) defined by the experimental set-up to which one must add the reaction coming from \( \mathcal{V} \) itself; strictly, both \( V \) and \( \mathcal{V} \) are quantized and coupled together but as a first approach, and with some caution, a semi-classical approximation can be used. The reaction is given by a mean value \( \lambda \langle 0 | O(t) | 0 \rangle \), where \( O(t) \) is some observable of the particle, \( \lambda \) a small coupling constant and \( | 0 \rangle \) the vacuum state at \( t = 0 \); one gets

\[
V(t) = V^{\text{ext}}(t) + \lambda \langle 0 | O(t) | 0 \rangle ,
\]

where \( V^{\text{ext}} \) is small and adiabatic, the reaction reduces to a vacuum polarization and can often be neglected, but it becomes important as soon as \( V^{\text{ext}} \gg m \). To see what happens when \( V^{\text{ext}} \) is switched on amounts then to

1) finding a solution of the Cauchy problem between \( t = 0 \) and \( t \) for the linear equation (1) and calculating

\[
\langle 0 | O(t) | 0 \rangle = F \langle \mathcal{V}(t) \rangle ,
\]

2) satisfying \( V(t) = V^{\text{ext}}(t) + \lambda F \langle \mathcal{V}(t) \rangle \).

This is difficult mostly because \( F \) is nonlinear relatively to \( V \).

With this approach particle creation is always a scattering process, nevertheless we can define the supercritical process as those particles created in the adiabatic limit.

A simplification occurs if we take into account that, the closure of the gap happens by coalescence of two isolated points of the spectrum (for instance, in the heavy-ion experiment studied in ref. (2), the external field comes not only from a high-Z positive nucleus, but also from \( Z \) negative charges producing a part of the point spectrum just above the lower continuum).

This is not the case in our model, but it suffices to take \( \mathcal{V} \) in a box of size \( L \) with the \( \mathcal{V} = 0 \) boundary condition to model it.

Canonical quantization gives then a discrete set of operators \( a_n(t) \), \( n = 0, 1, 2 ... \) (the usual annihilation operators) the first has frequency \( \kappa_0 = (m - V(t))^2 \), all other have frequencies \( \kappa_0^2 + n^2 / L^2 \), so that the breakdown of adiabaticity for \( V^{\text{ext}}(t) \) slow, occurs in the \( a_0(t) \)'s equation first and the back-reaction can be calculated from the Cauchy problem for \( a_0(t) \) only.