Conformal Relativity, a Theory of Mass.

I: Survey of Theoretical Results (*).

R. L. INGRAHAM

Research Center, New Mexico State University - Las Cruces, N. Mex. 88003.

(ricevuto il 21 Novembre 1977)

Summary. — In a series of papers it will be explained how a kinematic theory of mass follows from adopting the conformal group $\mathcal{C}$ as basic space-time symmetry group. As an aspect of the group, kinematic masses are bare masses; physical masses are to be calculated from these and interactions. The particle solutions are of two kinds: massless, or massive families consisting of the $\mathcal{C}$-multiplet plus an infinite set of discrete excited mass states. An unexpected consequence of conformal symmetry is the extrapolation of the space-time origin of (some at least) internal symmetries. In particular, $\mathcal{C}$-multiplets provide isospin labels and the free Lagrangians, pure conformal group constructs, possess isospin $SU_2$ invariance as an accidental (or dynamical) symmetry. New possibilities for gauge theories of interactions are also implied. The new way of handling mass allows gauge bosons of nonzero bare mass (as well as massless ones) without violating gauge invariance. Thus Higgs fields and vacuum broken symmetries are no longer necessary in gauge theory.

1. — Introduction.

Conformal relativity is the extension of ordinary relativity in which mass is treated on an equal footing with momentum and energy. That is, just as the latter yields a kinematic theory of free-particle momentum and energy, an aspect of the symmetry group itself, so does the former yield in addition a kinematic theory of free-particle bare mass. By conformal relativity we

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.
shall mean the theory in which the conformal group \( \mathcal{C} \) is substituted for the Poincaré group \( \mathcal{P} \) (inhomogeneous Lorentz group) as the basic space-time symmetry group of the world.

Other new features which flow from the same source are a changed picture of causal signal propagation at very small distances (nonzero minimum time lags), the elucidation of the purely space-time origin of (some at least) internal symmetries, and a theory relating certain coupling constants. This is the first of a series of several papers in which we propose to justify these claims.

First, let us take a look at the status quo in relativistic physics. On the basis of special relativity free-particle momentum and energy, united into 4-momentum, and their conjugate co-ordinates space and time, are well understood. We know how they are related (1):

\[
\begin{align*}
p^2 + m^2 &= 0, \\
p^2 &= p^2 - E^2,
\end{align*}
\]

we know precisely how they transform and mix up as we change to another inertial frame, we know their spectra. This theory is all kinematics, i.e. the consequences of defining the various particle species as the IURs (irreducible unitary representations) of \( \mathcal{P} \).

But the fifth kinematical quantity, \( m \) in eq. (1), remains outside, unexplained. One might have hoped that the basic symmetry group, here \( \mathcal{P} \), would predict the spectra of all kinematical quantities as they are actually observed, the mass spectrum as well as the 4-momentum spectrum. This reasonable expectation is thwarted because \( m \) happens to be an invariant of \( \mathcal{P} \), for which the values \( m = 0 \) or \( 0 < m < \infty \) are allowed, with no indication by the theory of which values actually occur. Nature actually seems to require a limited set of massless particles plus (possibly infinite) "families" of massive particles whose masses proceed in more or less regular steps to infinity (2).

It seems amazing that we have come so far, to such sophisticated theories (3) and to such a degree of confidence in our imminent understanding of the weak, EM (electromagnetic) and strong forces (4), while totally lacking this basic ingredient, a theory of mass.

Certainly current physics talks about and predicts various masses, but these calculations are typically of the following types. Given some bare masses

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(1) We shall use natural units \( \hbar = c = 1 \). The metric is \( g_{\mu\nu} = \text{diag}(+ + + -) \).

(2) This speculation concerns bare (unrenormalized, undressed) masses. Also we mean observed particles, whether they be elementary or composite.

(3) For example, the gauge theories of unified weak, EM and strong forces in the last ten years.

(4) Cf. "The Key to the Universe", a television broadcast co-produced by the BBC and WTTW, Chicago.