Determination of the Nucleus-Nucleus Potential from the Generalized Critical-Distance Model.

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Summary. – In the present work we get an analytical expression of the nuclear part \( V_N \) of the nucleus-nucleus potential in terms of the critical-distance and entrance-channel parameters. The critical distance is obtained in the framework of the generalized critical-distance model. By using \( V_N \), we derive for several reactions the universal function \( U(s) \), which displays the known scaling behaviour.

In the last years many works have been devoted to investigate the properties of the nucleus-nucleus potential \(^{(1,4)}\). In the framework of such investigations, we determine an analytical expression of the nuclear part \( V_N \), of the nucleus-nucleus potential in terms of the critical-distance and entrance-channel parameters. The critical distance is obtained in the framework of the generalized critical-distance model. By using \( V_N \), we derive for several reactions the universal function \( U(s) \), which displays the known scaling behaviour.

Let us remind that in the sharp cut-off approximation for the transmission coefficients, the fusion cross-section can be written as

\[
\sigma_t = \lambda^2 l_c^2,
\]

where \( l_c \) is the critical angular momentum, \textit{i.e.} the highest value of angular momentum

which contributes to fusion; moreover, from the energy conservation condition we have

\begin{equation}
V_c(r_e) + \frac{\hbar^2 l_e (l_e + 1)}{2 \mu r_e^2} + V_N(r_e) = E,
\end{equation}

where \( V_c(r_e) = (Z_1 Z_2 e^2)/r_e \) and \( V_N(r_e) \) is the nuclear potential at \( r = r_e \).

From eq. (1) and eq. (2) we get

\begin{equation}
\alpha_i = \pi r_e^2 \left( 1 - \frac{V_c(r_e) + V_N(r_e)}{E} \right)
\end{equation}

or

\begin{equation}
E \alpha_i = \pi r_e^2 \left[ E - (V_c(r_e) + V_N(r_e)) \right].
\end{equation}

By differentiating eq. (4) with respect to energy, one obtains

\begin{equation}
\frac{d}{dE} (E \alpha_i) = \pi r_e^2 \left[ \frac{d}{dr} \left( \pi r^2 (E - V_c(r) - V_N(r)) \right) \right] \frac{dr_e}{r_e}.
\end{equation}

On the other hand, we have

\begin{equation}
\frac{1}{\pi} \frac{d}{dr} \left\{ \pi r^2 [E - V_c(r) - V_N(r)] \right\} = 2r \left( E - V_c(r) - V_N(r) \right) + r^2 \left( \frac{d}{dr} V_c(r) - \frac{d}{dr} V_N(r) \right),
\end{equation}

and if the effective potential attains its maximum in \( r = r_e \), i.e.

\begin{equation}
\left[ \frac{d}{dr} \left( V_c(r) + V_N(r) + \frac{\hbar^2 l_e (l_e + 1)}{2 \mu r^2} \right) \right]_{r=r_e} = 0,
\end{equation}

then

\begin{equation}
\left[ \frac{d}{dr} V_N(r) \right]_{r=r_e} = - \left[ \frac{d}{dr} V_c(r) \right]_{r=r_e} + \frac{\hbar^2 l_e (l_e + 1)}{2 \mu r_e^2}.
\end{equation}

By using eq. (8), we get from eq. (6)

\begin{equation}
\left[ \frac{d}{dr} \left\{ \pi r^2 [E - V_c(r) - V_N(r)] \right\} \right]_{r=r_e} = 0,
\end{equation}

so that eq. (5) becomes

\begin{equation}
\frac{d}{dE} (E \alpha_i) = \pi r_e^2.
\end{equation}

Under the hypotheses of the generalized critical-distance model (5,6), i.e.: