Perturbative Corrections in Superconductivity. - II

R. K. Colegrave and G. D. Koukoutas

Department of Mathematics, Chelsea College, University of London - London SW3 6LX

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In our previous communication (1) we investigated two classes of diagrams in an attempt to calculate some perturbative corrections to the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity. By simulating the interaction by a modified shell-model potential we were able to perform calculations for a) the second-order diagrams and b) all ring diagrams, and to compare the order parameter $\Delta$ and the transition temperature $T_c$ with the BCS values. Appreciable changes were found to result from the inclusion of the second-order diagrams, but a partial summation over ring diagrams led to fractional changes of order $10^{-4}$ only. The purpose of the present work is to calculate the effect on $T_c$ of all the ladder diagrams, both direct and exchange. Such a partial summation has been discussed by Nieminen (2) and should have a greater physical significance than our previous calculations.

As before, we assume a spin-independent potential

$$V_{k-k'} = \begin{cases} -\bar{V} \text{ (constant)}, & \text{for } |k-k'| < q_0, \\ 0, & \text{otherwise}, \end{cases}$$

which corresponds to an attraction over a range of order $10^{-4}$ cm. The renormalized pair propagator is

$$G(k, \omega) = \hbar [\hbar\omega - (\epsilon_k - \mu) \tau_3 - \Sigma(k, \omega)]^{-1},$$

where $\Sigma$ is the sum of all the irreducible self-energy diagrams. We couple this with the Gor’kov equation for the energy gap $\Delta$, which may be assumed real:

$$G(k, \omega) = \hbar [\hbar\omega - (\epsilon_k - \mu + Z_k) \tau_3 - J\tau_1]^{-1},$$

where $\chi_k$ is the Hartree-Fock potential and $\tau_1$, $\tau_2$ are Pauli matrices. Following Schrieffer (3) we put $\chi_k = 0$ and then eqs. (2) and (3) lead to

$$A = \Sigma_{\omega}(k, \omega) = \Sigma_{\omega}(k, \omega).$$

We follow the Nambu-Gor'kov rules and evaluate the sum of ladder diagrams. The lowest-order diagrams will be found to give the BCS result, which forms a useful check on our working. Thus, if we employ a four-component notation for the elevated temperature case, $k = (k, i\omega_n)$, $k' = (k', i\omega_m)$.

$$-i\Sigma(k) = \Gamma(kk', kk') + \Gamma(k'k, kk').$$

On using the Poisson formula for the summation over discrete frequencies, the direct and exchange kernels are given by

$$(6a) \quad \Gamma(kk', kk') = -V1 + \int \frac{d^3k}{(2\pi)^3} \int \frac{d\alpha}{2\pi} f(\alpha) \tau_3 \langle G(k - q, i\omega_n - \alpha)(-iV_q) \tau_3 \langle G(k + q, i\omega_m + \alpha) \cdot \tau_3 \Gamma(k - q, i\omega_n - \alpha; k' + q; i\omega_m + \alpha; k, i\omega_n; k', i\omega_m),$$

$$(6b) \quad \Gamma(k'k, kk') = V_{k-k'}1 + \int \frac{d^3q}{(2\pi)^3} \int \frac{d\alpha}{2\pi} f(\alpha) \tau_3 \langle G(k' + q, i\omega_m + \alpha)(-iV_q) \tau_3 \langle G(k - q, i\omega_n - \alpha) \cdot \tau_3 \Gamma(k' + q, i\omega_m + \alpha; k - q, i\omega_n - \alpha; k, i\omega_n, k', i\omega_m),$$

where $f(\alpha)$ is the Fermi distribution function and the contour $C$ encloses the imaginary $\alpha$-axis.

We shall abbreviate our notation and write $\Gamma(kk')$, $\Gamma(k'k)$ with matrix elements $\gamma_{ij}(kk')$, $\gamma_{ij}(k'k)$, respectively. Also, since the cut-off $q_0$ in momentum transfer is small, we need to make only a first iteration of eqs. (6), leading to

$$(7a) \quad \Gamma(kk') = -V \left( \begin{array}{cc} 1 + \bar{V}_\chi(k, k') & 0 \\ 0 & 1 + \bar{V}_\eta(k, k') \end{array} \right),$$

$$(7b) \quad \Gamma(k'k) = V_{k-k'} \left( \begin{array}{cc} 1 + \bar{V}_\chi(k', k) & 0 \\ 0 & 1 + \bar{V}_\eta(k', k) \end{array} \right).$$