Inelasticity and Partial-Wave Dispersion Relations.

E. J. Squires (*)
Tait Institute of Mathematical Physics - Edinburgh

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Summary. — A problem consisting of two coupled channels is considered, and it is shown that the use of a one-channel N-over-D method, with the inelasticity introduced through the factor $R$ in the unitary relation $\text{Im} \alpha = R |\alpha|^2$, does not necessarily give the correct solution.

These comments are wholly inspired by a preprint of J. Rothleitner and B. Stech (University of Heidelberg) entitled *Is the Nucleon a Bound State?* (to be published in *Zeitschrift für Physik* and referred to here as R.S.). We shall endeavour to show that the elementarity or otherwise of the nucleon is not in question in the discussion of R.S., but that their result can be reinterpreted to imply that the standard «no free parameters» N-over-D method need not give the correct physical solution when there is inelasticity.

To explain this, let us first consider the $s$-wave partial-wave amplitude in a single-channel problem. We assume that the input force (i.e., the left-hand cut) is given, and is such that there is a bound state, and a phase shift which behaves as shown by the solid line in Fig. 1. The fact that the phase shift tends to $-\pi$ as $v \to \infty$ follows from Levinson’s theorem, and the important point about Fig. 1 is that the phase shift, and hence the amplitude, passes through zero at $v = \bar{v}$.

It is convenient to have a simple soluble problem in mind, and for this purpose we can consider the problem where the left-hand cut is replaced by 

(*) Future address: Department of Mathematics, University of Durham, Durham.
two poles (1). In this case (*)

\[ N = \frac{\Gamma_1 D(-v_1)}{v_1 + \nu} + \frac{\Gamma_2 D(-v_2)}{v_2 + \nu} \]

and

\[ D(\nu) = 1 - \frac{\Gamma_1 D(-v_1)}{\sqrt{\nu + \sqrt{v_1}}} - \frac{\Gamma_2 D(-v_2)}{\sqrt{\nu + \sqrt{v_2}}}, \]

where \( \Gamma_1 \) and \( \Gamma_2 \) are the residues of the poles, and \( v_1 \) and \( v_2 \) are their positions (\( v_1 \) and \( v_2 \) are both positive). These two equations uniquely determine the amplitude, and suitable choice of the input parameters \( v_1, v_2, \Gamma_1 \) and \( \Gamma_2 \) can yield a bound state at \( \nu = \nu_* \) and a phase shift of the form of Fig. 1.

We now modify the problem by introducing a small coupling to a second channel with threshold at \( v_* < \tilde{v} \). Since the coupling is small, it is plausible that it has little effect on the elastic amplitude in the first channel (\( a_{11} \), say). It does, however, have a relatively important effect in the neighbourhood of the region where, without the coupling, the elastic amplitude becomes zero, \( i.e., \nu \approx \tilde{v} \). Indeed, it is easy to see from unitarity,

\[ \nu^{-1} \text{Im} a_{11} = |a_{11}|^2 + |a_{12}|^2, \quad \nu > \nu_1, \]

that \( a_{11} \) is unlikely to pass through zero in the physical region above the inelastic threshold, since this would require both the elastic and inelastic \( (a_{12}) \) amplitudes, to be zero simultaneously. We ignore this possibility which would be accidental. A consequence is that the phase of the amplitude \( a_{11} \) will not pass through zero. This follows (see R.S.) from the expression for the phase \( \varphi \) in terms of the real part \( (x) \) and imaginary part \( (\beta) \) of the phase shift, \( i.e., \)

\[ \text{tg} \varphi = \frac{1 - \exp[-2\beta] \cos 2x}{\exp[-2\beta] \sin 2x}. \]

For small coupling, \( x \) is approximately equal to the (real) phase shift \( (\delta) \) for zero coupling, and passes through zero close to \( \nu = \tilde{v} \). However, unless \( \beta = 0 \) at the same point, \( \varphi \) does not pass through zero, but behaves as shown by the dotted line in Fig. 1.

These considerations can readily be confirmed by means of the \( N \)-over-\( D \) matrix equations for the two-channel problem where the left-hand cuts are

(1) We take nonrelativistic kinematics, although this is immaterial for our purposes here.