Summary. — The effective coupling constants \( f_{\gamma\pi\gamma} \) and \( f_{\eta\pi\gamma} \) are calculated from first principles by using single-variable dispersion representations. Baryon-antibaryon intermediate states are taken to dominate the calculation, and the reasons for this are discussed. We consider separately the special case in which vector mesons dominate the baryon electromagnetic form factors, and the general case, in which they do not. In the general case, meaningful equations can result only when the pion, alone, is taken off the mass shell, and then only if one subtraction is made at infinity. A convergence condition must be imposed on the integrand; the result is determined by this condition alone, and consequently depends only on the high-energy behaviour of the spectral function. In the special case, either the pion or the photon can be taken off the mass shell, both choices leading to the same result. On the assumption of vector dominance, this circumstance reconciles a result of Feldman and Matthews with a model of Gell-Mann, Sharp and Wagner. The results can be evaluated only provisionally, for lack of reliable input data on the strong interactions; such results as do emerge conform to experiment. The same methods are used to calculate the dependence of the \( \pi^0 \rightarrow \gamma e^+ e^- \) amplitude on the squared total mass of the Dalitz pair, i.e. of the virtual \( \gamma \). In contrast to previous approaches, we find that in general a once-subtracted dispersion relation must be used; the subtraction constant must be supplied by an independent calculation of \( f_{\gamma\pi\gamma} \) in which only the pion leaves the mass shell. Our result (like previous ones) contradicts experiment as regards the magnitude of the dependence and, less unambiguously, as regards its sign; but, short of abandoning very basic theoretical ideas, there remains much less scope than formerly for reconciliation with experiment. A remeasurement is recommended.
1. – Introduction.

This paper has three interrelated aims. The most immediate one is to estimate, by means of a direct dynamical calculation, the effective V-P-γ coupling constants $f_{\gamma \gamma \gamma}$, where V and P symbolize vector and pseudoscalar mesons respectively. We call our approach "dynamical" because it complements other lines of argument, which try to correlate several amplitudes without necessarily calculating the absolute size of any of them.

The second aim is to clarify the assumptions underlying such calculations on three-point functions in general. Our tools are the single-variable dispersion representations. The philosophy is to take from experiment all the needed parameters (and phase shifts where relevant) characterizing the strong interaction, and to regard these as the input data of the calculation; the electromagnetic amplitudes, calculated to leading order in $e$, form the output \(^{(1)}\). The input data are accepted without reservations; in practice not all have been measured reliably, and many can be procured only by making ad hoc assumptions which, logically, do not form part of the theory. Hence, except for orders of magnitude, the theory can be confronted with observation at best provisionally. The input data are discussed again at the beginning of Sect. 5, and in the Appendix.

As in all such calculations, one must approximate the spectral functions of the dispersion integrals by restricting the contributory intermediate states to a manageable few. Throughout this paper, we shall include only the baryon-antibaryon states, called BB in the following. The reason for this is explained in Sect. 4.2. (In one special case the single-V states will also be included, for reasons which the context will make obvious.)

Two problems of principle arise at once; both are absolutely central to the whole subject. First, which squared mass should be selected as the dispersion variable (i.e., which of the three particles V, P, or γ should be taken off the mass shell) \(^{(2)}\); and second, how many subtractions are needed in the dispersion relation that is selected? It will become obvious in the following that in general the three alternative dispersion relations involve quite different combinations of the input data. Hence, if more than one of them lead to a meaningful result, we shall be faced either with more than one theoretical value for $f_{\gamma \gamma \gamma}$, without any criterion for choosing between them; or else with

\(^{(1)}\) We make one exception by treating the anomalous magnetic moments of the baryons (and indeed the baryon electromagnetic form factors) as input. It will appear that this is strongly suggested by the structure of the equations themselves.

\(^{(2)}\) There is no such ambiguity in the original problem of charged pion decay \(^{(3)}\), nor in the boson pole approximation to $\gamma \to 3\pi$ decay \(^{(4)}\); these are governed by two-point functions, each with a single "propagating" momentum.