Quantum Mechanics and High-Energy Physics.

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**Summary.** - We discuss the phenomenological implications of a recent suggestion that the canonical commutation relations of quantum mechanics should be generalized to be applicable in the high-energy domain. Some evidence is given that this modification could become important around a mass of $10^8$ GeV typical of partial grand unification schemes.

It has been suggested, recently, that the canonical commutation relations of quantum mechanics could retain some validity in the high-energy limit if properly generalized. The suggestion (1) is to replace $[q_i, p_i] = i$ by

$$[q_i, p_i] = i + i\hat{f}(H),$$

where $\hat{f}(H)$ is some Hermitian operator function of the Hamiltonian $H$ (this guarantees that $[q_i, p_i]$ remain constants of motion). Natural units $\hbar = c = 1$ are used in this paper. For (1) to reduce to the canonical commutation relations, one requires the expectation value of $\hat{f}(H)$ to vanish at low energies. No further constraint is *a priori* obvious on $\hat{f}(H)$ and a tentative linear form is explored in ref. (1) as the simplest possible one.

While it is by no means obvious that the basic principles of quantum mechanics should retain any validity at high energy, this is an intriguing possibility worth being considered. In this spirit and assuming eq. (1) to be a realistic generalization of the usual canonical commutation relations, it seems to us that the present high-energy data

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should be used to narrow down the arbitrariness of $f(H)$ and to determine, at least very roughly, at which energies important modifications should be expected. It is quite interesting that our considerations will lead us to a functional form of $f(H)$ which is basically the same suggested in ref. (1) (up to logarithmic corrections).

We begin by assuming that the uncertainty relation which follows from (1) holds, at least approximately, with an equality sign

$$\Delta q_i \Delta p_i \simeq \frac{1}{2} f(\sqrt{s}),$$

where $f(\sqrt{s})$ is the expectation value of $f(H)$.

Suppose now we apply eq. (2) to a proton in a $S^\infty$ system, i.e. in a system in which the proton is moving at a very large energy (transverse variables only need be considered). Although some discrepancy exists among the various collaborations working at the CERN p$\bar{p}$ collider, recent data confirm the indications from cosmic rays that $\Delta p_\perp = \sqrt{\langle p_\perp^2 \rangle}$ grows with energy (2). Although it is much too soon to say which empirical form of growth will better reproduce the data, these are not in conflict with the QCD-inspired form (3)

$$\langle p_\perp \rangle \simeq \alpha + \beta (\sqrt{s} - M) \ln (\sqrt{s}/M),$$

where $M$ is some appropriate low-energy mass at which $\langle p_\perp \rangle$ becomes strictly constant. We shall take $M \simeq 1 \text{ GeV}$.

If we now consider the phenomenological implications for $\Delta p_\perp$ identified with the impact parameter $b$, i.e. with the radius of the proton $R(s)$, this is known to be a function of $\sqrt{s}$ increasing with an empirical form (4)

$$\Delta p_\perp \simeq R(s) \propto \ln (\sqrt{s}/M).$$

Combining eqs. (2)-(4) we are led to the qualitative indication

$$f(\sqrt{s}) \propto (\sqrt{s} - M)(\ln \sqrt{s}/M)^2,$$

i.e. to a form which is compatible with a linear dependence of $f(H)$ on $H$ (up to logarithms). It is noteworthy that a linear form for $f(H)$ is exactly the form advocated in ref. (1) and it is quite remarkable that such a linear dependence is indeed suggested by the high-energy data.

Specializing eq. (1) to meet the requirement implied by (5), we now have

$$[q_i, \pi_i] = i + iL(H - M) \ln^2 (H/M),$$

where $L$ is a parameter with the dimension of a length for which we shall now try to give a rough estimate.

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