Nonlinear Photons in the Universe.

II - The Anisotropic Case

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(ricevuto il 22 Febbraio 1984)

PACS. 78.80. – Cosmology.

Summary. – We examine the consequences of nonminimal coupling of electromagnetic and gravitational fields. An exact solution is presented, which corresponds to a generalization of the anisotropic universe of Kasner and which can be interpreted as a previous era of an isotropic (photon dominated) Friedmann-like universe.

Classical configurations of energy distribution corresponding to long- and/or short-range fields coupled minimally to gravitation belong to the class of entities which generate singular cosmologies.

This is an almost direct consequence of the so-called singularity theorems of classical general relativity.

So, in the search of a nonsingular universe we have been led to investigate the properties of nonminimal coupling of electromagnetic and gravitational fields.

Our theory starts with the Lagrangian

\[ L = \sqrt{-g} \left[ \frac{1}{k} R - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - R W_{\mu} W^{\mu} \right] \]

in which conventions are as usual (see (1)).

In (1) we have found a new solution of the equations derived from Lagrangian (1) which leads to some fascinating features of the Universe.

Among those, we can quote that this cosmos has no singularity, it has no particle horizon and contains a unique free parameter (the longitudinal electromagnetic field)

which allows one to fix the density of highest compression of the cosmos (which separates a contracting phase from our expanding era) (1).

Here we intend to present a new exact solution of equations derived for (1) which is a sort of generalization of the standard anisotropic Universe found by Kasner.

The equations of motion are (2)

\[
(2a) \quad \left[ 1 - \frac{1}{k} W^2 \right] \left[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right] = -E_{\mu\nu} + \Box W^2 g_{\mu\nu} + W^2, W_{\mu} W_{\nu} + R W_{\mu} W_{\nu},
\]

\[
(2b) \quad f_{\mu\nu} = RW_{\mu},
\]

in which \( E_{\mu\nu} \) is Maxwell's tensor

\[
(3) \quad R_{\mu\nu} = f_{\mu\alpha} f_{\nu} + \frac{1}{2} g_{\mu\nu} f_{\alpha\beta} f^{\alpha\beta}
\]

and

\[
W^2 = W_{\mu} W_{\nu} g_{\mu\nu}.
\]

We see from (2b) and taking the trace of (2a) that the nonminimal coupling (represented by the mass \( \rho \) of the photon) induces nonlinearities in the equation of motion of the electromagnetic field.

Let us now look for a cosmical solution of this set of equations which represents a homogeneous and anisotropic universe.

We set

\[
(4) \quad ds^2 = dt^2 - a^2(t) dx^2 - b^2(t) dy^2 - c^2(t) dz^2
\]

and

\[
W_{\mu} = (W(t), 0, 0, 0).
\]

Equation (2a), (2b) reduce in this case to the set

\[
(5a) \quad \ddot{a} + \ddot{b} + \ddot{c} + \ddot{\gamma} = 0,
\]

\[
(5b) \quad \ddot{a} + \ddot{b} + \ddot{c} + \ddot{\gamma} = 0,
\]

\[
\frac{\ddot{b}}{b} + \frac{\ddot{b}}{b} \left( \frac{\dot{a}}{a} + \frac{\dot{c}}{c} + \frac{\dot{\gamma}}{\gamma} \right) = 0,
\]

\[
\frac{\ddot{c}}{b} + \frac{\ddot{c}}{b} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{\gamma}}{\gamma} \right) = 0,
\]

\[
\frac{\ddot{\gamma}}{\gamma} + \ddot{\gamma} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} \right) = 0,
\]

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(\dagger) \quad M. NOVELLO and H. HEINTZMANN: An Eternal Universe, to be published.
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