The Relationship of Second-Type Singularities
to Normal Thresholds at the Edge of the Physical Region.

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Summary. — We show that the discontinuity across the two-particle threshold in a subenergy variable of a two-particle-three-particle amplitude possesses a second-type singularity. This singularity has the important property of determining on which sheet of their two-particle thresholds the integrated subenergies should be taken in the integral giving the two-particle discontinuity. This singularity also has the property of complicating the sheet structure of the two-particle subenergy thresholds and the three-particle threshold in the total energy in such a way that it is not obvious that the three-particle threshold can be isolated. However, we show that it is possible to find a class of paths which essentially just encircle the three-particle threshold and no other singularity. We then show that the second-type singularity plays a vital role in the derivation of Olive's discontinuity formula for the three-particle threshold.

1. — Introduction.

The physical-region singularities of the $S$-matrix required by unitarity have been shown to lie on Landau curves and general methods for deriving the discontinuities across these singularities inside the physical region have been developed (1). A natural extension of this work is to singularities occurring at the edge of the physical region. In this paper we consider normal thresholds in the total and subenergy variables of a 2-3 amplitude (i.e. an amplitude for the production of three particles from two), occurring at the edge

of the physical region for this amplitude. We consider particles of one mass
only and show that in this case the discontinuity across the two-particle thresh-
old in a subenergy variable possesses a second-type singularity \((13)\) at another
boundary of the physical region. This singularity is not a physical-sheet sing-
gularity of the amplitude but plays a vital role when the two-particle discon-
tinuity is analytically continued out of the physical region in order to evaluate
the discontinuity of the amplitude across the three-particle cut in the total
energy.

The discontinuity of an amplitude across a physical region singularity is
obtained essentially by comparing the unitarity equations holding on either
side of the singularity. One equation is analytically continued around the sin-
gularity to the region where the other equation holds. The same method may
be used for singularities at the edge of the physical region if an unphysical
unitarity equation holding on the unphysical side of the singularity can be ob-
tained. BOYLING \((1)\) has shown how such unphysical unitarity equations may
be obtained for the amplitude we are considering using an extension of Olive's
method for proving Hermitian analyticity, \(i.e.\) using the existence of poles
in the physical region of multiparticle amplitudes, and the factorization of
their residues. However, BOYLING was not able to completely justify all of
the analyticity assumptions needed for his proof, and we must still regard such
unphysical unitarity equations as an assumption.

Initially we consider the two-particle threshold in a subenergy variable.
LANDSHOFF, OLIVE and POLKINGHORNE \((3)\) have derived the discontinuity
across such a threshold where because particles of different mass are intro-
duced the threshold under study occurs inside the physical region. If we con-
sider particles of one mass only, then the only two-particle threshold in a par-
ticular subenergy will be at the edge of the physical region. However, if we
use the unphysical unitarity equation holding below the threshold, the work
of LANDSHOFF, OLIVE and POLKINGHORNE can be applied directly to give

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\begin{align*}
\text{+} & - \text{+} = \text{+} \text{+} - \\
\end{align*}
\]

\(\text{(1)}\)

The notation is Olive's \((4, 5)\) except that to indicate that a particular subenergy

\(\text{(13)}\) R. J. EDEN, P. V. LANDSHOFF, D. I. OLIVE and J. C. POLKINGHORNE: The


7, 1593 (1966).


\(\text{(5)}\) R. J. EDEN, P. V. LANDSHOFF, D. I. OLIVE and J. C. POLKINGHORNE: The