On the $SU_3$ Breaking Factor (*).

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Summary. - It is shown that the $SU_3$ breaking factor in the Field-Feynman quark jet model may be fixed or correlated to other parameters by the momentum sum rules, if the primary mesons under consideration do not constitute whole nonets.

It is well-known that the parton fragmentation at virtualness $Q^2$ can be viewed as a two-step process: The parton at $Q^2$ transforms into other partons with much lower virtualness $Q'_a$, which then fragment into hadrons. The first step can be treated (1) perturbatively under QCD, while the second step belongs to the nonperturbative part. The nonperturbative quark fragmentation function, denoted by $D^h_q(x)$ with the hadron $h$ carrying the momentum fraction $x$ of the quark, is expressed in terms of two unknown functions and a set of parameters under the Field-Feynman (2) model. One of the above parameters is the $SU_3$ breaking factor, denoted by $\gamma_a/\gamma_\nu$, where $\gamma_a$ is the probability of creating $q\bar{q}$ pair. Usually (3), it is considered as a free parameter to be determined experimentally. In this note, we consider only three quark flavours and want to show that if the primary mesons under consideration do not constitute whole nonets, then the $SU_3$ breaking factor may be fixed or correlated to other parameters by the momentum sum rules.

In the Field-Feynman quark jet model, the fragmentation functions can be written as

\begin{equation}
D^h_q(x) = \delta_{qa} \gamma_b f(1 - x) + \gamma_a \gamma_b \bar{f}(x),
\end{equation}

where $a$ and $b$ represent the quark flavours, $\gamma_a$ is the probability of creating an $a\bar{a}$ pair and $f(1 - x) (\bar{f}(x))$ represents the probability density of finding the rank-one (any rank larger than one) primary meson with momentum fraction $x$ of the initial quark. By means of expression (1), all the fragmentation functions can be expressed in terms

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of $f$, $\bar{F}$ and $\gamma$'s:

$$D^h_q(x) = \sum_b C_{ab} \gamma_b f(1-x) + \sum_{ab} \gamma_a \gamma_b C_{ab} \bar{F}(x),$$

where $C_{ab}$ represents the probability of the meson $h$ at the state $ab$. In this note, we consider only three quark flavours, therefore we have

$$2\gamma + \gamma_s = 1,$$

where $\gamma = \gamma_u = \gamma_d$.

As mentioned above, we have assumed that the nonperturbative processes which cause hadronization start at $Q_0^2$. But how large is $Q_0^2$? There is no definite answer to it. Since the value of $Q_0^2$ will limit the production of the primary mesons owing to the kinematical reason, we will consider various cases corresponding to a different choice of $Q_0^2$. In the following, we will demonstrate how the momentum sum rules for various cases restrict the value of $\gamma$.

**Case 1.** The primary mesons consist of only $\pi$ and $K$ mesons. By using expressions (2) and (3), the momentum sum rule

$$\sum_b \int_0^1 dz zD^h_q(z) = 1$$

becomes

$$\int_0^1 dz z\left[\left(1 - \frac{\gamma}{2}\right)f(1-z) + (4\gamma - 5\gamma^2)\bar{F}(z)\right] = 1,$$

for $q = u$ and $d$. For the $s$-jet, the sum rule (4) becomes

$$\int_0^1 dz z[2\gamma f(1-z) + (4\gamma - 5\gamma^2)\bar{F}(z)] = 1.$$

Expressions (5) and (6) then imply

$$\left(1 - \frac{5}{3}\gamma\right)\int_0^1 dz zf(1-z) = 0.$$

Since the integral is nonvanishing, we have $\gamma = \frac{8}{3}$. Therefore, the consistency between momentum sum rules implies the $SU_3$ breaking factor $\gamma_s/\gamma$ is equal to $\frac{1}{3}$.

**Case 2.** The primary mesons consist of $\pi, K, \eta, \varphi, K^*$ and $\omega$ mesons. The quark contents of $\rho$ and $\omega$ are chosen to be

$$\eta = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}),$$

$$\omega = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}).$$