Dirac-Like Form for the Linear Equations for the Gravitational Field.

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Linear gravitational field equations, that are obtained from Maxwell's equations by using the gravitational vector \( G \) and \( H \) instead of the electric \( E \) and the induction \( B \), have attracted the attention of several authors \(^{(1,2)}\). When Heavisidian monopoles are taken into account the equations can be modified by introducing two scalar fields \( G_t \) and \( H_t \). The quaternion form of the equations in this modified version was obtained by Singh \(^{(3)}\). The fields \( G_t \) and \( H_t \) were introduced by Ohmura \(^{(4)}\) in the electromagnetic theory.

It is the aim of the present letter to obtain the Dirac-like form for the changed equations. The procedure to be followed is similar to the one used in the case of the electromagnetic theory \(^{(5)}\). Expressions for the Clifford aggregates which are the wave function and the current are indicated. As expected \( iG_t \) and \( iH_t \) are fourth components of vectors whose three first components are \( G_k \) and \( H_k \) \((k = 1, 2, 3)\), respectively. When \( G_t \) and \( H_t \) are not found to be necessary the equations are simplified. The forms of the wave function and of the current are indicated in this case.

The linear gravitational field equations are

\[
\begin{align*}
\nabla \cdot G + \frac{1}{c} \frac{\partial G_t}{\partial t} &= -4\pi q, \\
\nabla \cdot H + \frac{1}{c} \frac{\partial H_t}{\partial t} &= -4\pi q', \\
-\nabla \times H + \frac{1}{c} \frac{\partial G}{\partial t} + \nabla G_t &= (4\pi/c) j, \\
\n\nabla \times G + \frac{1}{c} \frac{\partial H}{\partial t} + \nabla H_t &= (4\pi/c) j',
\end{align*}
\]


where $G$ and $H$ are gravitational vector fields, whereas $G_t$ and $H_t$ are scalar fields. With the help of the relation

$$(\sigma \cdot a)(\sigma \cdot b) = a \cdot b + i\sigma \cdot a \times b,$$

it is easy to see that (1) can be written as

$$\left[\begin{array}{cc}
\frac{\sigma}{c} & 1 \\
-1 & 0 \end{array}\right] \frac{\partial}{\partial t} + \left[\begin{array}{c}
0 \\
0 \end{array}\right] \cdot \nabla \left[\begin{array}{c}
\sigma \cdot G + iH_t \\
\sigma \cdot H - iG_t \end{array}\right] = \frac{4\pi}{c} \left[\begin{array}{c}
i\sigma \cdot j - cG' \\
-i\sigma \cdot j + cG' \end{array}\right].$$

The wave function and the current of eq. (2) can be identified as elements of left-minimal ideals as follows:

\begin{itemize}
  \item[i)]
  \begin{align*}
  \psi & \rightarrow \psi_I = \begin{bmatrix} \sigma \cdot G + iH_t & 0 \\
  \sigma \cdot H - iG_t & 0 \end{bmatrix}, \\
  J & \rightarrow J_I = \begin{bmatrix} i\sigma \cdot j - cG' & 0 \\
  -i\sigma \cdot j + cG' & 0 \end{bmatrix}
  \end{align*}

\item[ii)]
  \begin{align*}
  \psi & \rightarrow \psi_{II} = \begin{bmatrix} 0 & \sigma \cdot G + iH_t \\
  0 & \sigma \cdot H - iG_t \end{bmatrix}, \\
  J & \rightarrow J_{II} = \begin{bmatrix} 0 & i\sigma \cdot j - cG' \\
  0 & -i\sigma \cdot j + cG' \end{bmatrix}.
  \end{align*}
\end{itemize}

If

$$\gamma_4 = \begin{bmatrix} 1 & 0 \\
0 & -1 \end{bmatrix}, \quad \gamma_k = \begin{bmatrix} 0 & \sigma_k \\
\sigma_k & 0 \end{bmatrix} \quad (k = 1, 2, 3)$$

is the representation of the Dirac matrices to be used, it can shown that (3a), (3b), (4a) and (4b) can be written as

\begin{align*}
  \psi_I & = \frac{1}{2} (\gamma_4 G_{\mu} + i\gamma_5 \gamma_\mu G_{\mu})(1 + \gamma_4), \\
  J_I & = \frac{1}{2} \gamma_4 (i\gamma_\mu j_\mu - \gamma_5 \gamma_\mu j'_\mu)(1 + \gamma_4),
\end{align*}

and

\begin{align*}
  \psi_{II} & = \frac{1}{2} (\gamma_4 G_{\mu} - i\gamma_5 \gamma_\mu H_{\mu})(1 - \gamma_4), \\
  J_{II} & = \frac{1}{2} \gamma_4 (i\gamma_\mu j_\mu + \gamma_5 \gamma_\mu j'_\mu)(1 - \gamma_4).
\end{align*}

In (6a), (6b), (7a) and (7b), $G_\mu = G_k$ for $\mu = 1, 2, 3$ and $G_\mu = iG_t$ for $\mu = 4$. The same holds for $H_\mu$. 

