Effect of Elastic Anisotropy on the Nonlinear Optical Properties of a Nematic Cell (*).

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Summary. – We show for the first time the effect of the elastic anisotropy on the nonlinear optical behaviour of a nematic liquid crystal and point out the enhancement of it due to an increase of the elastic anisotropy.

In recent publications we have pointed out the interest of a hybrid aligned nematic cell to have strong nonlinear optical properties (1) and we have reported the first complete theory (2) of optically induced molecular reorientation in nematic liquid crystals, applying it to a hybrid configuration. In that paper we were able to predict the nonlinear optical response $\langle \delta \varepsilon \rangle$ for any applied field. This theory drops all the approximations previously adopted by other authors, in particular different values of the elastic constants $K_1$ and $K_3$ are allowed. In this way, it is possible for the first time to make an analysis of the influence of the elastic anisotropy on the nonlinear optical properties of a nematic cell.

In this letter we report the results of computer calculations of $\langle \delta \varepsilon \rangle$ for different values of the elastic anisotropy $K = 1 - K_1/K_3$, while other typical parameters are kept constant. The aim of this study is to point out that the nonlinear optical properties are very sensitive of this anisotropy in a hybrid aligned cell.

In the following we will recall briefly the theory upon which calculations are based and then we will discuss the results.

A hybrid nematic sample has different molecular orientation of the limiting walls, e.g. homeotropic on the first glass and planar on the second one. If the anchoring energies on the boundaries are finite, the pretilt angles on the walls $\varphi_1$ and $\varphi_2$ (3) are different

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(3) $\varphi$ is measured with respect to the normal to the boundary.
from 0 and $\pi/2$, respectively, and have to be found from the boundary conditions; moreover, they change as the bulk orientation angle $q$ does when the optical field is varied. Actually $q = q(z, A^{-1})$, where $z$ is the co-ordinate along the cell thickness and $A^{-1}$ is proportional to the field amplitude.

We recall only the results of the theory leading to the following three equations needed to calculate $q_1$, $q_2$ and the integration constant $c^2$:

1. $$2L_i((1-K\sin^2q_i)\left[(c^2-f(q_i, A))\right] = \sin 2q_i$$
   $$\begin{cases} i = 1 \text{ for } z = -\frac{d}{2} \quad i = 2 \text{ for } z = \frac{d}{2} \end{cases}$$

2. \[ \int_{q_i}^{q_f} \left\{ \frac{1-K\sin^2q}{c^2-f(q, A)} \right\} \, dq = d, \]

where $d$ is the thickness of the cell and the following relationships are valid:

$$f(q, A) = A^{-2}(2\epsilon_1, \Delta\epsilon_1) \left[ 1 + \left( \frac{\Delta\epsilon}{\epsilon} \right)^2 \right] \cos^2 q,$$

$$L_i = k_i/W_i$$

with $W_i$ as anchoring energy on the walls, $\epsilon_1$ and $\epsilon_\perp$ the dielectric constants parallel and perpendicular to the molecular director. $|A|$ is the amplitude of the optical field.

In this way it is possible to know the actual deformation expressed by $q(z, A^{-1})$ through the following integral equation:

$$\int_{q_i}^{q_f} \frac{1-K\sin^2q}{c^2-f(q, A)} \, dq = d.$$ 

This step is necessary to calculate the nonlinear dielectric constant from the expression

$$\langle \delta\epsilon(z, A) \rangle = \frac{\Delta\epsilon}{2} - \frac{\left( \Delta\epsilon \right)^2}{2} \left( \frac{\cos 2q(z, \infty) - \cos 2q(z, A)}{\epsilon + (\Delta\epsilon/2) \cos 2q(z, A)} \right) \left( \epsilon + (\Delta\epsilon/2) \cos 2q(z, \infty) \right),$$

where $\langle \rangle$ means averaging along the cell thickness.

When we compute the function $\langle \delta\epsilon \rangle$ we get new interesting information about the role of the elastic anisotropy on the nonlinear optical response of a hybrid nematic cell.

Calculations have been made using $\epsilon_1 = 3.2$, $\epsilon_\perp = 2.4$, $D = 10\mu m$ and varying $K$ from $-0.9$ to $0.9$.

In fig. 1 is considered the case of different anchoring energies on the boundaries ($L_1 = 1\mu m, L_2 = 10\mu m$). Only three curves are reported for the sake of clearness, but calculations were made for many intermediate values of $K$. Figure 2 shows the strong anchoring case ($L_1 = 0\mu m, L_2 = 0\mu m$).