Some Properties of a Hamiltonian Model of Broken $SU_3 \times SU_3$ Symmetry (*)

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The significance of the $SU_3 \times SU_3$ symmetry has recently been clarified greatly by the works of GLASHOW and WEINBERG (1), and of GELL-MANN, OAKES and RENNER (2). In particular, according to GOR, the strong-interaction Hamiltonian density can be written in the explicit form

\begin{align}
\hat{H} &= \hat{H}_0 + \hat{H}', \\
\hat{H}' &= \alpha (u_0 + \sqrt{2} \tau u_0),
\end{align}

where $\hat{H}_0$ is invariant under $SU_3 \times SU_3$, and $u_i$, $i = 0, ..., 8$, together with $\tau$, are scalar (pseudoscalar) densities belonging to the $(3, \bar{3}) + (3, 3)$ representation of $SU_3 \times SU_3$. It was argued by GOR that the constant $\alpha$ should take approximately the value $-1$. It has since been emphasized by CABIBBO and MAIANI (3) that $\hat{H}'$ is by no means unique. Indeed, since $\hat{H}_0$ is invariant under $SU_3 \times SU_3$, if $\hat{H}' = R \hat{H}' R^{-1}$, where $R$ is any rotation in the $SU_3 \times SU_3$ space, then physically the two theories described by $\hat{H} = \hat{H}_0 + \hat{H}'$ and $\hat{H} = \hat{H}_0 + R \hat{H}'$ are equivalent. The equivalence of $\hat{H}$ and $\hat{H}'$ is established thru redefinitions of the hadron internal quantum numbers, as well as the physical state vectors. The question arises as to the uniqueness of the constant $\alpha$ defined in eq. (2). In other words, is it possible to find a rotation $R$ such that $\hat{H}' = \tilde{\alpha} \cdot (u_0 + \sqrt{2} \tau u_0)$, and if so, what relation there is between $\alpha$ and $\tilde{\alpha}$? In this work we wish to give an explicit $R$ which does that, and to discuss some of the consequences of the nonuniqueness of $\alpha$.

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(2) M. GELL-MANN, R. OAKES and B. RENNER: Phys. Rev., 175, 2195 (1968). This paper will be referred to as GOR.
We shall use the notation of CM where, corresponding to the expression
\[ \sum (a_i u_i + b_i v_i), \]
one defines the 3×3 matrix \( \mathcal{M} \) by
\[ \mathcal{M} = \sum (a_i + ib_i) \lambda_i. \]

Under a rotation \( R \), let \( U \) and \( V \) denote the 3×3 rotation matrices in the \( SU_{3L} \)
and \( SU_{3R} \) spaces, then the transformation law of \( \mathcal{M} \) is
\[ \mathcal{M} \rightarrow V^\dagger \mathcal{M} U, \]
\[ \mathcal{M} \rightarrow U^\dagger \mathcal{M}^+ V. \]

In this notation, \( H' \) as defined in eq. (2) can be represented by a matrix of the form
\[ \mathcal{H}' = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix}, \]
where \( a \) and \( b \) are real numbers, so that \( H' \) does not contain any \( v_i \)'s. Physically, this
corresponds to the conservation of isospin, hypercharge, as well as the parity operation
defined by
\[ P(u_i, v_i) P^{-1} = (u_i, -v_i). \]

In this work we shall assume that the directions in \( SU_3 \) space are already known
through the medium strong \(^{(4)}\), the electromagnetic and the weak interactions. This means
that, given any matrix \( \mathcal{H}' \), we should always make a rotation to bring it into the diagonal
form, which is then regarded as the physical frame of reference.

Our problem is then to find \( U \) and \( V \) such that
\[ \mathcal{H}' = V^\dagger \mathcal{H}' U, \]
\[ \mathcal{H}' = \begin{pmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{a} & 0 \\ 0 & 0 & \tilde{b} \end{pmatrix}, \]
where \( \tilde{a} \) and \( \tilde{b} \) are real numbers. Since the directions in \( SU_3 \) space are assumed known,
\( U \) and \( V \) commute with the isospin and hypercharge operators. They can therefore
only be rotations around the 8th axis in the \( SU_{3L} \) and \( SU_{3R} \) spaces. The rotations
generated by \( F_8 \) are trivial, while those generated by \( F_8^6 \) violate \( P \), in general, i.e.
the \( \mathcal{H}' \) obtained according to eq. (8) is complex. However the finite rotation
\[ W = \exp \left[ -\frac{3\pi}{2} \cdot Y^5 \right], \]

\(^{(4)}\) Actually it is only necessary to require that the direction of the electromagnetic interaction be
fixed. This point, as well as other details of our work, will be discussed further in a forthcoming paper.