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Recently, the perturbation treatment of like nucleons in a large shell has been developed (1) without explicit use of any c.f.p. (2). In this note we show that a similar treatment is possible to study the configuration mixing in the low-lying states of the s.c.s. (single-closed shell) nuclei expressed, in the zeroth-order approximation, by the configuration $j^n$ with seniority $v$ less than 3. Our study is essentially an application of reduction relations (1-3) about a square sum of c.f.p. over the additional quantum number. The result will provide a powerful physical insight, since all the quantities to evaluate will be shown in simple algebraic form which can be applied for any large spin $j$.

Let us consider the effect of the mixed configurations on the expectation values of a two-body interaction, noting that the diagonal matrix element of the unperturbed state with $j^v v < 2$ is expressed (4) in terms of two-body matrix elements only. For simplicity, we discuss only the configurations that differ by a single-nucleon orbit from the configuration $j^n$.

Firstly, we can show for the general $v$ the following complete (so long as the additional quantum number is kept) reduction relations by use of the reduction relation (3) about

$$\sum \langle j^v v \varphi | j^v \varphi | j^v v \varphi >$$

(for simplicity, only the case of $j_1 \neq j$ is considered here):

$$\langle j^v v \varphi | V \varphi | j^v - 1 \varphi' j_1 j > = \frac{n - v}{2j + 1 - 2v} \sqrt{2j + 3 - n - v} \langle j^v v \varphi | V \varphi | j^v - 1 \varphi' j_1 j > +$$

$$+ \frac{2j + 1 - n - v}{2j + 1 - 2v} \sqrt{2j + 3 - n - v} \langle j^v v \varphi | V \varphi | j^v - 1 \varphi' j_1 j > ,$$

\begin{align}
(2) \quad \langle j^n v \alpha J | V | j^{n-1} (v + 1 \alpha' J') j_1 J \rangle &= \\
&= (-)^{l_1 + l'_1 - j_1 - j_2} \frac{2j + 1 - n - v}{2j - 1 - 2v} \sqrt{\frac{(J')(n - v)}{[J](2j + 1 - 2v)}} \langle j^{n+1} v + 1 \alpha' J' | V | j^n (v \alpha J) j_1 J' \rangle + \\
&\quad + (-)^{l_1 + l'_1 - j_1 - j_2} \frac{n - v - 2}{2j - 1 - 2v} \sqrt{\frac{(J')(n - v)}{[J](2j + 1 - 2v)}} \langle j^{n+1} v + 1 \alpha' J' | V | j^n (v \alpha J) j_1 J' \rangle ,
\end{align}

\begin{align}
(3) \quad \langle j^n v \alpha J | V | j^{n-1} (v - 3 \alpha' J') j_1 J \rangle &= \\
&= \sqrt{(n - v + 2)(2j + 3 - n - v)(2j + 5 - n - v)} \langle j^v v \alpha J | V | j^{n-1} (v - 3 \alpha' J') j_1 J \rangle ,
\end{align}

\begin{align}
(4) \quad \langle j^n v \alpha J | V | j^{n-1} (v + 3 \alpha' J') j_1 J \rangle &= \\
&= (-)^{l_1 + l'_1 - j_1 - j_2} \sqrt{(n - v - 2)(2j + 1 - n - v)} \langle j^{n+1} v + 3 \alpha' J' | V | j^{n+2} (v \alpha J) j_1 J' \rangle ,
\end{align}

Here, \([J] = 2J + 1\), the particle-hole interaction \(\bar{V}\) is defined by

\begin{align}
(5) \quad \bar{V}_J &= \langle j^2 J | \bar{V} | j_1 J \rangle = -2 \sum_K \begin{bmatrix} j & j & K \\ j & j_1 & J \end{bmatrix} \langle j^2 K | V | j j_1 K \rangle ,
\end{align}

with the antisymmetrized and normalized wave function \(|jj_1 J\rangle\), and we adopt the phase convention that is consistent with ref. (2). Note that matrix elements of \(V\) and \(\bar{V}\) satisfy

\begin{align}
(6) \quad \sum_J [J] \bar{V}_J^2 &= 2 \sum_J [J] V_J^2 - \sum_J [J] V_J \bar{V}_J ,
\end{align}

where \(J\) runs over even integers. Note also that the matrix element

\begin{align}
\langle j^{n-1} v \alpha J | V | j^n (v \alpha' J') j_1^{-1} J \rangle
\end{align}

is related to the left-hand side of (1)-(4) by

\begin{align}
(7) \quad \langle j^{n-1} v \alpha J | V | j^n (v' \alpha' J') j_1^{-1} J \rangle &= (-)^{l_1 + l'_1 - j_1 + 1} \sqrt{\frac{[J']}{[J]}} \langle j^n v' \alpha' J' | V | j^{n-1} (v \alpha J) j_1 J' \rangle .
\end{align}

We can show for the general \(v\) another type of reduction relations

\begin{align}
(8) \quad \langle j^n v \alpha J | V | j^n (v \alpha' J') j_1 J_0 J \rangle &= \frac{\langle j^n v \alpha J | \bar{U}^{(J_0)} | j^n v \alpha J \rangle}{\langle j^n v \alpha J | \bar{U}^{(J_0)} | j^n v \alpha J \rangle} \langle j^n v \alpha J | V | j^n (v \alpha' J') j_1 J_0 J \rangle
\end{align}