S-Wave Kaon-Pion Scattering.

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(received on 19 July 1965)

Summary. — The inverse amplitude method is used to find iterative self-consistent solutions for S-wave kaon-pion scattering. The experimental values are employed for the K* resonance and the p-meson, in addition to one-pole approximation for the l = 0, T = 0 pion-pion amplitude, given by Hamilton, representing the ABC anomaly. Crossing symmetry for the real part of the scattering amplitude and a sum rule for $A^{(+)}(s, u, t)$ enable the results to be expressed in terms of only one parameter, $\alpha_0^{(+)}$. Numerical results are given for negative values of this parameter.

1. — Introduction.

The kaon-pion interaction plays an important role in processes involving kaons, such as kaon-nucleon scattering. The force of longest range comes from the exchange of a pion-pair in the isotopic spin state $T = 0$ (1). Such an exchange takes place through the reaction: $\pi\pi \rightarrow KK \rightarrow N\bar{N}$. A knowledge of the kaon-pion, pion-pion and pion-nucleon scatterings is necessary to determine this process.

In this paper we try to solve the low-energy kaon-pion scattering problem. No attempt is made to determine self-consistently the K* resonance. A more complicated approach is necessary for this involving the coupling of two channels (2). Assuming the $p$-wave scattering amplitude in the isotopic spin state $I = \frac{3}{2}$ and all the higher partial waves to be very small in the low energy region, we consider the two S-wave amplitudes and the K* resonance.

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The necessary kinematics are discussed in Sect. 2. The method of the inverse amplitude (1) is used in Sect. 3 to write down partial-wave dispersion relations for the $S$-wave scattering amplitudes. In the complex $S$-plane, very high energy scatterings contribute through crossing around the point, $S = 0$. Now the square of the magnitude of the three-momentum, $K^2$ behaves like $K^2 \to \pm \infty$ as $S \to \pm 0$. Thus by considering $A^{t-1}_0(s)/k^2$ instead of $A^{t-1}_0(s)$ these high-energy contributions are suppressed. The inverse amplitude dispersion relation in a particular isotopic spin state for this quantity involves two subtraction constants. One of these is connected with the scattering length and the other is connected to the value of the scattering amplitude at the crossed threshold, $S = (m - \mu)^2$. Using crossing symmetry the real part at the crossed threshold may be expressed in terms of the scattering lengths for the two isotopic spin states. A sum rule is written down connecting the scattering lengths for the isotopic spin states. If the discontinuities across the unphysical cuts can be calculated then it is possible to determine the low energy $S$-wave scattering amplitudes depending on only one arbitrary parameter. This is chosen to be the combination, $a_0^{(+)}/a_0 = \frac{1}{2}(a_0^2 + 2a_0^2)$ of the scattering lengths.

In the contributions of the crossed kaon-pion channel to the discontinuities across the unphysical cuts only the $K^*$ resonance using the experimental results and the $S$-wave amplitudes arising out of the iteration described in Sect. 3 are retained. To calculate the contributions of the third channel, which describes the process: $\pi\pi \to \bar{K}K$, to the unphysical cuts we have to determine the latter process. This is done for the low-energy region in Sect. 4. The determination of this process, in turn requires the knowledge of the pion-pion scattering amplitudes. Luckily, the latter is a closed process. Only the contributions of the ABC anomaly in the $I = 0, \ J = 0$, state and the $q$ resonance in the $I = 1, \ J = 1$ state are retained for the pion-pion amplitudes. On the left hand cut of the $\pi\pi \to \bar{K}K$ channel only the contributions coming from the $K^*$ resonance and the $S$-wave kaon-pion scattering amplitudes arising out of the iteration process are retained.

A very brief discussion of the iteration process is given in Sect. 5. The numerical results obtained are also discussed.

2. - Kinematics.

We are interested in the following three reactions:

I) $\pi(p_1, \alpha) + K(p_2) \to \pi(- p_3, \beta) + K(- p_4)$,

II) $\pi(p_2, \beta) + K(p_3) \to \pi(- p_1, \alpha) + K(- p_4)$,

III) $\pi(p_1, \alpha) + \pi(p_2, \beta) \to \bar{K}(- p_3) + K(- p_4)$.