Multichannel Potential Model of $\pi N$ Scattering
in the $I = \frac{1}{2}, J = \frac{1}{2}^+$ State (*).

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Summary. — A multichannel-potential model, which incorporates some relativistic kinematics, correct threshold behaviors, space-time symmetries and coupling to channels containing unstable particles is developed. The model is a generalization of the separable-potential model of Yamaguchi. It is applied to $\pi N$ scattering in the $I = \frac{1}{2}, J = \frac{1}{2}^+$ state. The $\pi N$ and $\sigma N$ (where $\sigma$ is an unstable $I = 0, J = 0^+$ two-pion state) channels are taken into account. The parameters of the model are adjusted to give 1) a nucleon bound state at the physical nucleon mass. 2) a zero in the $\pi N$ amplitude at a pion laboratory kinetic energy of 150 MeV, 3) the $\pi N$ Roper resonance at a pion laboratory kinetic energy of 600 MeV and 4) the experimental inelasticity parameter. We then find the $\pi N$ coupling constant $g^2_{\pi N}/4\pi = 15.2$ and the $\pi N$ $I = \frac{1}{2}, J = \frac{1}{2}^+$ scattering length $a_{\pi N} = -0.06$ in units of the cube of the pion Compton wavelength, in fair agreement with the experimental values $g^2_{\pi N}/4\pi = 15.1 \pm 0.006$ and $a_{\pi N} = -0.101 \pm 0.007$, respectively. The $\sigma N$ coupling constant $g^2_{\sigma N}/4\pi$ was found to be $\sim 7$ in comparison with estimates of $\sim 4 \div 5$ from the analysis of nucleon-nucleon scattering.

1. — Introduction.

The matrix $N/D$ method (1) has been the most widely used method for treating relativistic multichannel scattering problems. This method, however, generally leads to a rather complicated set of integral equations unless ap-

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proximations are introduced (2). Because of our present limited knowledge of
the dynamics of strongly interacting particles, it appears likely that all formal-
isms which yield unitary scattering amplitudes with correct space-time sym-
metries, threshold behaviors, relativistic kinematics, explicit dynamical effects
of known long-range forces and parametrized effects of unknown short-range
forces will have roughly the same validity.

In an attempt to introduce a formalism somewhat simpler than that of the
$N/D$ method, a multichannel generalization of the Yamaguchi separable po-
tential (3-4) was recently proposed (5). This formalism can give scattering
amplitudes with the properties mentioned above and, because of its simpli-
city, may be useful for gaining insight into the behavior of systems containing
large numbers of coupled channels. The amplitudes given by the formalism
are somewhat similar to those coming from pole approximations for the $N/D$
method (6).

In Sect. 2 of this paper, we extend the model of ref. (5) to include spi-
interactions in arbitrary angular-momentum states, and channels containing
unstable particles. In Sect. 3, we apply the formalism to $\pi N$ scattering in the
$I = \frac{1}{2}$, $J = \frac{1}{2}^+$ state with the $\pi N$ and $\sigma N$ (being an unstable $I = 0$, $J = 0^+$
two-pion state) channels being taken into account. A summary and some con-
cluding remarks are contained in Sect. 4.

2. - Multichannel separable potential model.

2'1. Two-particle states. - Single-particle states,

$|K, \nu\rangle$

are defined in terms of their four-momentum

$K (= K_\nu)$,

$K_\nu = (K^0, K) = (E, K)$

$K^0 = E = \sqrt{\mu^2 + (K)^2}$,


(6) For the amplitude of J. S. Ball and M. Parkinson: Phys. Rev.,
162, 1509 (1967), eq. (2), would essentially follow from the eqs. (2.11)-(2.14) of ref. (5)
if $g_{\alpha\nu}(k_\alpha) = \delta_{\alpha\nu} g_\nu(k_\alpha)$. 

(2) See e.g. A. W. Martin: Phys. Rev., 162, 1534 (1967) for references to various
approximation schemes.


(4) For an extensive literature on recent applications of separable potentials, see