The Two-Channel Model of the $\pi$-Meson.

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Summary. — The pion model in terms of the two channels, $\pi + N\bar{N}$, is discussed, employing a matrix $N/D$ technique with pole approximation to the left-hand-cut contributions. The inclusion of the annihilation channel is shown to be of great importance. The dissociation probability of the pion into the $N\bar{N}$ state will be expected to be fairly small as compared with that into the $\rho\pi$ mode when the force in this channel is of the same order of magnitude as those in the elastic channels.

1. — Fermi and Yang were the first to consider the $\pi$-meson as a composite state (1). If the pion could be built from the nucleon-antinucleon system, it is necessary to suppose that there is a sufficiently strong short-range attraction to give the very large binding necessary for this bound state to correspond to the observed pion mass value. It seems unlikely that such an attractive force is generated by the exchange of bosons of low mass, and that exchange forces of similar kind are primarily concerned with the mass spectra of hadrons. Thus, in working with nearby-singularity contributions which are generally considered as the best understood, one has to give up the pion-mass problem.

In the usual pion models, the $N\bar{N}$ system has been studied as if this channel were decoupled from the annihilation process. Some unfavorable features inherent in the pion model of a single $N\bar{N}$ channel were demonstrated in a previous paper (2).


We shall discuss here the pion model in terms of the two channels, $\rho\pi$ and $NN$; the former is an approximate two-body system of the lightest state, $3\pi$, and the latter a familiar state of high mass. The pion mass is assumed to be given by yet unknown subtle mechanisms, for example, by «constructive» force. It is the principal aim to estimate the contributions from the $NN$ and $\rho\pi$ modes to LSZ's unitarity sum rule (3).

In the following Section we formulate the present problem in terms of Green's functions. Section 3 contains the application of the $N/D$ equations to the channel-coupling model with pole approximations to nearby dynamical singularities. The last Section contains a discussion of the results.

2. - We make a kinematical discussion of the pion model in terms of the $\rho\pi$ and $NN$ systems, specified by channels 1 and 2, respectively. The pion propagator is assumed to satisfy the unsubtracted Lehmann representation

\begin{equation}
\frac{i A_\rho(s)}{\mu^2 - s} = \sum_{i=1}^{2} \frac{1}{\pi} \int_{(m + \mu)^2}^{\infty} \frac{Q_i(s')|F_i(s')|^2}{(s' - s)(s' - \mu)^2} \, ds',
\end{equation}

where the phase-space factors are given by

\begin{equation}
\begin{cases}
Q_1(s) = \theta(s - (m + \mu)^2)4p^2s^3/m^2, \\
Q_2(s) = \theta(s - 4m^2)s(1 - 4M^2/s)^3,
\end{cases}
\end{equation}

and $p$ is the c.m. momentum of the $\rho\pi$ system, $F_i(s)$ the form factor for channel $i$ and $\mu$, $m$ and $M$ the masses of the $\pi$, $\rho$ and $N$, respectively. $Z_\pi(s)$ is defined by $i A_\rho(s)/i A_\rho(s)$ and expressed as

\begin{equation}
Z_\pi(s) = 1 + \sum_{i=1}^{2} \frac{s - \mu^2}{\pi} \int_{(m + \mu)^2}^{\infty} \frac{Q_i(s')|\Gamma_i(s')|^2}{(s' - s)(s' - \mu)^2} \, ds' + (s - \mu^2) \sum_{i=1}^{2} \frac{r_i}{\theta_i - s} \quad (r_i > 0)
\end{equation}

from the Helgolz property of the propagator; $Z_\pi = \lim_{s \to 0} Z_\pi(s)$ is identical to the wave renormalization constant in quantum field theory. LSZ's sum rule is obtained:

\begin{equation}
1 = Z_\pi + \sum_{i=1}^{2} \frac{1}{\pi} \int_{(m + \mu)^2}^{\infty} \frac{Q_i(s)|\Gamma_i(s)|^2}{(s - \mu^2)^2} \, ds + \sum_{i=1}^{2} r_i,
\end{equation}