An Associative Algebraic Model for Weak Meson Decays (*).

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(received il 24 Ottobre 1968)

Summary. — An algebraic model describing symmetry breaking is applied to the weak meson decay. The suppression of the strangeness-changing weak decay is obtained as a consequence of symmetry breaking.

In two previous papers (1,2) we have introduced a finitely generated associative algebra \( \mathcal{B} \) as a model for hadrons and obtained a meson mass spectrum in good agreement with the observed masses. The characteristic property of the algebra is that the generators \( H_i, E_{\pm a}, G_i, F_{\pm a} \) of the internal group \( SL_{2,\mathbb{C}} \) do not commute with the generators \( P_\mu \) of the Poincaré group \( \mathcal{P} \) but they do commute with the four-velocity operators \( M^{-1} P_\mu \). We are interested in extending the model to describe weak meson decays.

For this purpose we consider the possibility of factorizing the interaction in terms of a four-vector operator \( H_\lambda \) (hadronic part), which causes transitions between different components of the \( SL_{2,\mathbb{C}} \) multiplet and another four-vector \( I^\lambda \) which describes the leptonic part of the interaction but acts as an invariant with respect to the hadronic \( SL_{2,\mathbb{C}} \). The natural candidate for the operator \( H_\lambda \) is the operator

\[
H_\lambda = \sum_{\alpha = \pm 1, \pm 2, \pm 3} \{ P_\lambda, E_\alpha + F_\alpha \}.
\]

(*) Supported in part by the U.S. Atomic Energy Commission.
This operator is bilinear in the generators and is an element of the associate algebra $\mathcal{B}$.

This ansatz is suggested by the fact that the generators of $SL_3$ or $SU_3 \otimes SU_3$, $E_\alpha$ and $F_\alpha$, correspond to the integrals over the weak hadron currents. For the matrix element of $L^A$ we require that it describes the properties contained in conventional leptonic current. Then we consider writing an effective transition operator of the form

$$T = GL^A H_\lambda$$

(where $G$ is a constant characterizing the strength of the interaction) to describe the weak decay $\beta$. However the ansatz for the matrix element of $L^A$ has an unconventional form $(\beta)$ due to the following. It is in the spirit of this approach that $H_\lambda$ changes the energy-momentum of the states. Therefore—as the operator $T$ is to conserve energy-momentum—this is to be compensated in the leptonic part such that it results in over-all energy-momentum conservation. As $(2)$ is not in the usual form of a space-time integral of a suitable density this compensation requirement leads to the unconventional form for the matrix element of $L^A$.

To obtain our results in a more familiar way, we replace $(2)$ by

$$(2') T = G \int d^4x L^A(x) H_\lambda(x) ,$$

where $L^A(x)$ and $H_\lambda(x)$ are suitable field operators corresponding to $L^A$ and $H_\lambda$ respectively.

We want to consider the (semi)-leptonic decays

$$\beta \to \alpha_1 + \alpha_2 + \ldots + \alpha_N + l + \nu$$

where $\beta$ and $\alpha_1, \ldots, \alpha_N$ ($N = 0, 1$) are the various mesons and $l\nu$ is the lepton pair. Consequently we have to calculate

$$(4) \quad \mathcal{A} = \int d^4x \langle l\nu \alpha_1 \ldots \alpha_N \rho_l \rho_\nu \rho_{\alpha_1} \ldots \rho_{\alpha_N} | L^A(x) H_\lambda(x) | \beta p_\beta \rangle =$$

$$= \sum_\gamma \int \frac{d^3q_\gamma}{2E_\gamma} \int d^4x \langle l\nu \alpha_1 \ldots \alpha_N | L^A(x) | \gamma, q_\gamma \rangle \langle \gamma q_\gamma | H_\lambda(x) | \beta p_\beta \rangle .$$

The hadronic matrix element $\langle \gamma | H_\lambda(x) | \beta \rangle$ can be calculated in the frame of