A Model of Quantum Noise in Shadow Radiation Images

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Abstract—Correlation characteristics of a quantum noise in shadow X-ray images have been determined. A simple expression describing the structure of this noise has been suggested.

Shadow radiation images (RI) of tested objects (TO) are usually distorted owing to non-ideal properties of radiation sources [1–7]. Such distortions can be of different nature, and they are tentatively classified as systematic and random [1–4]. Systematic distortions are caused by the divergence of radiation beams and finite dimensions of sources. The apparent manifestation of these distortions is the fact that small features in the internal structure of a tested object are “fuzzy”, and the degree of this fuzziness depends on their positions between the source and radiation detector [3].

Random distortions of RI are due to the probabilistic character of both photon emission by sources and their interactions with materials of tested objects and detectors [7]. In X-ray photography, for example, such distortions are seen in the form of grains on films [6], which degrades the quality of photographs and their significance for detecting flaws.

Systematic distortions of RI and their quantitative estimates are discussed in detail elsewhere [2, 4]. Random distortions of RI were also analyzed in a number of publications [3, 4, 6–8]. Even so, the theoretical analysis of this issue should continue, which will broaden, beyond doubt, the field of application of X-ray techniques of nondestructive testing.

Mathematically, the problem in question is formulated as follows. The resulting RI,

\[ R(x, y, t) = N(x, y) + \eta(x, y, t) \]

is a sum of an ideal RI, \( N(x, y) \) and a random field (quantum noise), \( \eta(x, y, t) \). Our aim is to calculate correlation characteristics of the noise \( \eta(x, y, t) \), i.e., its mathematical expectation and correlation function. We define the ideal RI as an image not distorted by noise, namely the photon flux density distribution downstream of TO in the detection plane. In particular, in the absence of scattered radiation, systematic distortions, and the effect of random inhomogeneities in TO, we can use the expression [9]

\[ N(x, y) = N_0 \exp[-\mu H(x, y)][1 + \Delta \mu S(x, y)], \]

where \( N_0 \) is the photon flux density in the detector plane in the absence of TO, \( \mu \) is the linear attenuation factor (LAF) for a given material of TO, \( H(x, y) \) is the TO thickness, \( \Delta \mu = \mu - \mu_0 \), \( \mu_0 \) is the LAF of the material of an inhomogeneity in TO, and \( S(x, y) \) is the inhomogeneity dimension in the beam direction.

In order to solve the formulated problem, let us use the generally accepted assumption that the number of photons

\[ \xi = \int_0^T \int_\Omega R(x, y, t) \, dx \, dy \, dt, \]

incident on area \( \Omega \) in time \( T \) is a random value characterized by the Poisson distribution. It is known [10] that the mean value and dispersion of such random values are equal, i.e.,

\[ \xi = \sigma_x^2. \]

Equation (3) is the condition for calculating correlation characteristics of the noise \( \eta(x, y, t) \).
Taking into account (1) and (2), and also the general properties of the mathematical expectation, we calculate $\xi$ and $\sigma^2$. As a result of sequential calculations, we obtain

$$\overline{\xi} = \left\langle \int_0^T \int_0^\Omega R(x, y, t) \, dx \, dy \, dt \right\rangle = \int_0^T \int_0^\Omega \bar{R}(x, y, t) \, dx \, dy \, dt,$$

where $\bar{R}(x, y, t)$ is the mathematical expectation of $R(x, y, t)$.

Hereafter the symbol $\langle \rangle$, alongside the line over mathematical expressions, denotes the mathematical expectation. The averaged value of $\xi$ squared is calculated as follows:

$$\overline{\xi^2} = \left\langle \left[ \int_0^T \int_0^\Omega R(x, y, t) \, dx \, dy \, dt \right]^2 \right\rangle = \left\langle \int_0^T \int_0^\Omega R(x, y, t) \, dx \, dy \, dt \int_0^T \int_0^\Omega R(u, v, \tau) \, du \, dv \, dt \right\rangle,$$

$$= \int_0^T \int_0^\Omega \int_0^\Omega \left[ N(x, y) + \eta(x, y, t) \right] \left[ N(u, v) + \eta(u, v, \tau) \right] \, du \, dv \, dx \, dy \, dt,$$

$$= \int_0^T \int_0^\Omega \int_0^\Omega \int_0^\Omega \left[ N(x, y) N(u, v) + N(x, y) \eta(x, y, t) + N(x, y) \eta(u, v, \tau) + \eta(x, y, t) \eta(u, v, \tau) \right] \, du \, dv \, dx \, dy \, dt,$$

After substituting for convenience $\overline{\xi^2}$ in the form

$$\overline{\xi^2} = \left[ \int_0^T \int_0^\Omega \left[ N(x, y) + \eta(x, y, t) \right] \, dx \, dy \, dt \right]^2,$$

we obtain the dispersion of $\xi$:

$$\sigma_\xi^2 = \overline{\xi^2} - \overline{\xi^2} = \int_0^T \int_0^\Omega \int_0^\Omega \left[ \eta(x, y, t) \eta(u, v, \tau) - \overline{\eta(x, y, t) \eta(u, v, \tau)} \right] \, du \, dv \, dx \, dy \, dt.$$

These formulas can be used in applications where the mathematical expectation and the averaged value of $\xi$ squared are required.