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According to the asymptotic-scaling hypothesis of Koba, Nielsen and Olesen (KNO) (1) for the multiplicity distribution of charged secondaries in very-high-energy hadron-hadron collisions

\[ \frac{\sigma_n(s)}{\sigma_{\text{inel}}(s) s^{-\infty}} \approx \frac{1}{\langle n \rangle} \psi \left( \frac{n}{\langle n \rangle} \right) , \]

where \( \sigma_n(s) \) is the partial cross-section for producing \( n \) charged particles at the c.m. energy \( \sqrt{s} \), \( \sigma_{\text{inel}} \) is the total inelastic cross-section, \( \langle n \rangle \) is the average charged-particle multiplicity and \( \psi(x), x = n/\langle n \rangle \), is some function of \( x \) alone, independent of \( s \). According to KNO, this hypothesis is in line with Feynman's scaling law (2), and is strikingly supported by the data on pp collisions in the \((50 \div 300) \text{ GeV/c}\) energy range (3), the data on \( \pi^+p \) collisions in the \((10 \div 20) \text{ GeV/c}\) energy range (4), and also by \( \bar{p}p \) data in the \((3 \div 7) \text{ GeV/c}\) energy range (5).

It is also shown by KNO that asymptotically one has the ratio

\[ \frac{\langle n^q \rangle}{\langle n \rangle^q s^{-\infty}} \approx d_q , \]

\[ q = 2, 3, 4, \ldots \]

which is independent of energy. The \((\ln s)\)-dependence of \(\langle n \rangle\) seems to be satisfactorily
established, as has been shown by Antinucci et al. \(^{(4)}\). Another very important and
curious result comes from the Wroblewski formula \(^{(7)}\) according to which the dispersion
of \(n\) charged particles varies linearly with \(\langle n \rangle\)

\[
D = \sqrt{\langle (n - \langle n \rangle)^2 \rangle} = \sqrt{\langle n^2 \rangle - \langle n \rangle^2} = a \langle n \rangle - b,
\]

where \(a\) and \(b\) are positive constants. The value of \(b\) seems to be very small for the ISR
energy range and the stability of

\[
\frac{\langle n \rangle}{D} = (d_2 - 1)^{-\frac{1}{2}} = 2
\]

is reported for \(pp\), \(\bar{p}p\) and \(\pi^\pm p\) collisions \(^{(8)}\).

The linearity of the dispersion \(D\) (equivalently called the root-mean-square deviation of \(n\) from \(\langle n \rangle\)) with \(\langle n \rangle\) is a result which cannot be obtained by any single statistical
distribution. Thus Van Hove \(^{(9)}\), for example, considers two classes \(^{(10)}\) of inelastic
collisions (distributions), combines them binomially and by letting the probability of
one class of collisions to be very small \(^{(11)}\) obtains the desired result eq. (3). The
two-class idea of Van Hove is insufficient when one considers, for example, the third-
multiplicity distribution, where the dispersion is defined by

\[
D_3 = \langle (n - \langle n \rangle)^3 \rangle^{\frac{1}{3}},
\]

and when one attempts to get the generalized Wroblewski formula for this case, e.g.

\[
D_3 = c \langle n \rangle - d
\]

with \(c\) and \(d\) positive constants.

So, it is meaningful to consider, in the light of Van Howe's suggestion, that in order
to generate formula (5) one must consider at least three classes of inelastic collisions,
and in order to generate a generalized Wroblewski formula for \(q\)-th-multiplicity distri-
bution, with the dispersion defined by

\[
D_q = \langle (n - \langle n \rangle)^q \rangle^{\frac{1}{q}},
\]

one must consider \(q\) different classes of inelastic collisions at least.

Another serious difficulty seems to lie in finding the form of the scaling function
\(\psi(x), x = n/\langle n \rangle\). There is no unique function of \(x\) known so far to match \(\psi(x)\), although

\(^{(4)}\) Distributing \(n\) charged particles to two different classes is similar to the problem of flipping a coin
and finding the probability of the head or the tail coming up.
\(^{(11)}\) This corresponds, in the coin flipping problem, to a very low probability of either the head or the
tail coming up.