WEBER-LIKE INTERACTIONS AND ENERGY CONSERVATION

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Velocity-dependent forces varying as $k(\hat{r}/r)(1 - \mu r^2 + \gamma r\hat{r})$ (such as Weber force), here called Weber-like forces, are examined from the point of view of energy conservation and it is proved that they are conservative if and only if $\gamma = 2\mu$. As a consequence, it is shown that gravitational theories employing Weber-like forces cannot be conservative and also yield both the precession of the perihelion of Mercury as well as the gravitational deflection of light.

Key words: gravitational interaction, deflection of light, perihelion precession.

1. INTRODUCTION

One and a half century ago, when Weber [1] established the bases of his electrodynamics, the energy conservation arose as a central problem of the new theory since, for the first time, a velocity-dependent force law was stated for a basic interaction of nature:

$$ F_W = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} \hat{r} \left( 1 - \frac{1}{2c^2} \dot{r}^2 + \frac{1}{c^2} r \ddot{r} \right). $$ (1)

Here $q_1$, $q_2$ are the electric charges, $\epsilon_0$ is the vacuum permittivity, $r := ||r_1 - r_2||$, the separation distance from $q_2$ to $q_1$, and $\dot{r} := r/\dot{r}$; the dot signifies temporal derivation, and $c$ denotes simply the ratio between the electromagnetic and the electrostatics units of charge.

In order to face Helmholtz's criticism [2], Weber introduced for the first time a velocity-dependent potential energy

$$ U_W = \frac{q_1 q_2}{4\pi \epsilon_0 r} \left( 1 - \frac{1}{2c^2} \dot{r}^2 \right). $$ (2)
and succeeded to prove that \( \mathbf{F}_W \) is derivable from \( U_W \).

Some years later Tisserand [3] proposed a Weber-like gravitational force law

\[
\mathbf{F}_T = -\frac{Gm_1m_2}{r^2} \hat{\mathbf{r}} \left( 1 - \frac{1}{c^2 r^2} + \frac{2}{c^2 r \hat{r}} \right)
\]

derived from the Weber-like potential energy

\[
U_T = -\frac{Gm_1m_2}{r} \left( 1 - \frac{1}{c^2 r^2} \right),
\]

where \( m_1, m_2 \) are the gravitational masses, \( G \) is the gravitational constant, and \( c \) stands also for the speed of light. With this force, Tisserand obtained \( 3/8 \) of the then known value for the anomalous perihelion precession of Mercury and Levy [4], extending this potential energy, obtained the entire value for the precession.

In spite of its agreement with many theoretical and experimental results, Weber electrodynamics was replaced by the Maxwell-Lorentz field theory toward the end of the nineteenth century. And the interest in similar forces and potentials in gravitational theories also waned. Recently there has been a renewed interest in Weber electrodynamics in connection with important, but still controversial, experimental work [5, 6]. And there has been renewed interest in Weber-like interactions in gravitational theories, such as Assis’s Mach-like model [7]. With

\[
U_A = -\frac{Gm_1m_2}{r} \left( 1 - \frac{3}{c^2 r^2} \right)
\]

and

\[
\mathbf{F}_A = -\frac{Gm_1m_2}{r^2} \hat{\mathbf{r}} \left( 1 - \frac{3}{c^2 r^2} + \frac{6}{c^2 r \hat{r}} \right)
\]

Assis reobtained the correct expression for the perihelion precession.

More recently other theoretical Weber-like forces have been proposed to fit gravitational observations without, however, mentioning their conservative or nonconservative nature. Surprisingly enough, they are indeed generally nonconservative (as shown below). This means that the conservation of energy for Weber-like forces has not been adequately considered.

Raguza [8] extended Assis’s theory by proposing the force

\[
\mathbf{F}_R = -\frac{Gm_1m_2}{r^2} \hat{\mathbf{r}} \left( 1 - \frac{9}{c^2 r^2} + \frac{6}{c^2 r \hat{r}} \right),
\]