QUANTUM MEASUREMENT AND PURE DEPHASING

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The prerequisite of quantum measurement is a transformation of an initially off-diagonal density matrix $\rho_{m_\alpha; n_\beta}$ describing an interacting measured object and measuring device into a diagonal density matrix $\rho_{m_\alpha; m_\alpha}$, $\delta_{mn}$, $\delta_{\alpha\beta}$. The latter density matrix describes a proper mixture of states having definite $m$-values. On the other hand, the irreversible relaxation (towards the thermodynamic equilibrium) is also characterized by transformation of an initially off-diagonal matrix into a diagonal one. It has been shown that the process of irreversible relaxation can be used to perform quantum measurement, provided the duration $\Delta t$ of the measurement is much larger than $T_2$, the phase relaxation time, and much smaller than $T_1$, the population relaxation time: $T_2 \ll \Delta t \ll T_1$. A gedanken experiment describing this kind of measurement is provided. A $\pi/2$-pulse transforms an initial $s_z = -1/2$ state into superposition of $s_z = \pm1/2$ states. The irreversible relaxation leads to the proper mixture of $s_z = 1/2$ and $s_z = -1/2$ state. Results of the measurements are verified by the second electromagnetic pulse.

Key words: quantum measurement, wave-packet reduction, dephasing, density matrix.

1. INTRODUCTION

In a sense, the measurement process plays a central role in quantum mechanics (or, at least, in its interpretation). This problem continues to attract the attention and interest of researchers up until now (see, e.g., Refs. 1-4 and references therein). The problem is connected with the so-called wave-packet reduction, collapse of the wavefunction, and the Schrödinger cat paradox [1].
Suppose that we start with wave function

$$\psi \sum c_n u_n$$

and the corresponding density matrix

$$\rho_{mn} = c_n^* c_m.$$  \hspace{1cm} (1)

In the general case, we have a density matrix $\rho_{mn}$ with non-zero off-diagonal matrix elements. By measuring a dynamical quantity, say the energy of which $u_n$ are the eigenfunctions, we obtain various eigenvalues $E_1, \ldots , E_2, \ldots$ with probabilities $P_n = |c_n|^2$, or in the general case $P_n = \rho_{nn}$. But once we have obtained a given eigenvalue, say $E_n$, we know that the system is necessarily in the state $u_n$. At the end of the measurement process the initial wavefunction $\psi$ (density matrix $\rho$) is transformed into a mixture of various pure states $u_n$ with the probabilities $|c_1|^2, |c_2|^2, \ldots |c_n|^2, \ldots$, or $\rho_{11}, \ldots \rho_{22} \ldots \rho_{nn} \ldots$

The corresponding density matrix describing this proper mixture is

$$\rho = \sum \rho_{mn} |u_n\rangle \langle u_n|,$$

$$\rho_{mn} = |c_n|^2 \delta_{mn} \text{ or } \rho_{mn} = \rho_{nn} \delta_{mn},$$  \hspace{1cm} (3)

which is quite different from Eqs. (1) and (2).

The transformation from Eq. (1) to Eq. (3) often called the reduction of the wavepacket (or collapse of the wavefunction), does not belong to the type of unitary transformation described by the solution of the Schrödinger (von Neumann) equation.

On the other hand, considering the measuring device $M$ together with measured object $S$ as a whole united system described by the corresponding Hamiltonian, one may obtain that the object subsystem $S$ can be described by the diagonal density matrix. Therefore, it can be argued that the interaction of $M$ and $S$ changes system $S$ into a mixture of systems, a given proportion of which are in states that have definite $n$-values. The observer who looks at $S$ then simply acquires the knowledge of this already established situation.

This reasoning has been considered to be generally false [6]. The mixture of systems $S$ that emerges from the interaction could be an improper mixture. In other words, the fact that the density matrix of the subsystem $S$ is diagonal does not necessarily say that each system has a well-defined $n$-value. Thus, if the whole system $S + M$ had initially a wavefunction it will continue to be in a pure state (described by the wavefunction). It means the whole system $S + M$ is characterized by the corresponding density matrix having off-diagonal elements $\rho_{m\alpha \beta \gamma}$, while the subsystem $S$ may be described by diagonal density matrix

$$\sigma_{mn} = \sum \rho_{m\alpha \alpha \beta \gamma} \delta_{mn}.$$  \hspace{1cm} (4)