NUMERICAL ANALYSIS OF LAMINATED ANISOTROPIC SHELLS
AND PLATES BASED ON THE ITERATIVE SHEAR THEORY

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We consider a problem of bending of laminated anisotropic smooth shells and plates. On the basis of the method of iterations, we construct new, more precise mathematical models of the stress-strain state with different degrees of accuracy which take into account transverse shear strains. The properties of the developed models are studied by analyzing the problem of bending of rectangular diagonally reinforced laminated plates. It is shown that the application of the iterative approach to the construction of these models enables one to obtain results which converge to the three-dimensional solution of the problem as the degree of approximation of the model increases.

The analysis of laminated anisotropic shells and plates on the basis of the improved applied theories was performed in [1-4] and other works. At the same time, the investigation of the strain-stress state of these structures is complicated by the pronounced influence of transverse strains (mainly, of transverse shear strains).

In the present work, on the basis of the generalized iterative approach to the construction of two-dimensional mathematical models of physicomechanical processes [5], we develop an iterative shear theory, which enables one to obtain solutions with high degrees of accuracy. We construct an analytic solution of the problem of bending of anisotropic laminated rectangular plates. To check the validity of our results, we compare them with the three-dimensional solution presented in [6].

In a curvilinear orthogonal coordinate system \( O_{x_1,x_2} \), we consider a shell with anisotropic layers whose resistance to transverse shear strains is low (Fig. 1). Layers operate without separation and sliding. The material of the layers has a single plane of elastic symmetry. The coordinate axes \( x_1 \) and \( x_2 \) coincide with the lines of principal curvature of the coordinate surface \( x_0x_1 \) (the surface of reduction) and the axis \( x_3 = z \) is directed along the normal to this surface. The position of the surface of reduction across the thickness of the shell is chosen arbitrarily. For the coefficients of the first quadratic form, we assume that \( A_i = 1 \). The principal curvatures are supposed to be constant \( (k_i = \text{const}, i = 1, 2) \). The total thickness of the shell is regarded as small compared with the radii of curvature and, hence, \( 1 + k_i z = 1 \). The indices \( i, j = 1, 2 \) denote the directions of the coordinate axes \( x_i \) and \( x_j \), \( k \) is the number of a layer, and \( n \) is the total number of layers in the shell. Differentiation with respect to the corresponding variable is denoted by the comma placed in the subscript. Unlike the exponents, the superscripts are placed in parentheses.

For the problems of bending, the general iterative procedure can be formulated as follows [5]: in the \( M \)th step of iteration (the model of the \( M \)th approximation), the law of distribution of transverse tangential stresses is obtained by integrating equilibrium equations of the three-dimensional theory of elasticity with tangential components of the stress tensor established in the previous \( (M - 1) \)th step.

First, we deduce the relations of the classical theory based on the Kirchhoff–Love hypotheses. For the \( k \)th layer, these hypotheses can be written in the form

\[
\varepsilon_{i3}^{(k)} = 0, \quad \varepsilon_{3j}^{(k)} = 0, \quad \text{and} \quad \sigma_{ij} = 0. \quad (1)
\]
By using the first two hypotheses in (1) and the Cauchy relations, we obtain the following kinematic relations:

\[ u_i^{(k)} = v_i - w_i z, \quad u_3^{(k)} = w, \]

where \( u_i^{(k)}(x_j, z) \) and \( u_3^{(k)}(x_j, z) \) are tangential and normal displacements of the \( k \)th layer of the shell and \( v_i(x_j) \) and \( w(x_j) \) are tangential and normal displacements of the surface of reduction, respectively.

Relations (2) enable us to find the tangential components of the strain tensor \( e_{ij}^{(k)} \). Further, by substituting these components into Hooke's law, we determine the stresses \( \sigma_{ij}^{(k)} \). This completes the zero-order step of the procedure. It is now necessary to integrate the equilibrium equation for the \( k \)th layer

\[ \sigma_{ij,j}^{(k)} + \sigma_{j3,3}^{(k)} = 0 \]

by taking into account the conditions of contact of layers and the conditions imposed on the outer surfaces of the shell.

As a result, we obtain the expression for transverse tangential stresses in the form

\[ \sigma_{33}^{(k)} = - \int_{a_{k-1}}^{z} \sigma_{33}^{(k)} dz - \sum_{r=1}^{k-1} \int_{a_{r-1}}^{a_r} \sigma_{33}^{(k)} dz = - \int_{a_0}^{z} \sigma_{33}^{(k)} dz. \]

For the first step, relations (4) can be represented in the form

\[ \sigma_{33}^{(k)} = f_{331}^{(k)} x_{3i1} + f_{332}^{(k)} x_{2i1}, \]

where \( f_{331}^{(k)}(z) \) and \( f_{332}^{(k)}(z) \) are the laws of distribution of transverse tangential stresses across the thickness of the shell and \( x_{3i1}(x_j) \) and \( x_{2i1}(x_j) \) are the required functions of the surface of reduction (shear functions).

Transverse strains are determined by using Hooke's law with stresses given by (5). Then the procedure is repeated in exactly the same way as in the zero-order step. The kinematic relations for the model of the first approximation take the form

\[ u_i^{(k)} = v_i - w_i z - \psi_{3i1}^{(k)} x_{3i1} - \psi_{2i1}^{(k)} x_{2i1}, \quad u_3^{(k)} = w, \]

where \( \psi_{3i1}^{(k)}(z) \) and \( \psi_{2i1}^{(k)}(z) \) are the distribution functions of tangential displacements across the thickness of the shell.